U.T.B.M
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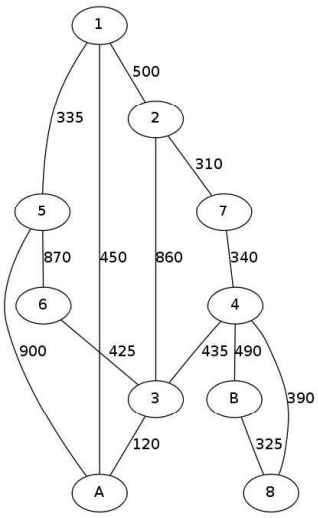
AG44

Final Exam - 2h

1 Maximum height (/)

The following network represents a portion of a city road network. All the roads are two-way and represented by the edges between two destinations. The numbers on the edges indicate the maximal heights (in centimeters) authorized for the vehicles. A driver wants to make a delivery from point A to point B. In such case, he wants to determine the maximal height x (in meters) of the truck which is authorized.

1. Show that this problem involves the resolution of a classical problem of graph. Which one ?
2. Quote and apply an algorithm to solve above problem.
3. Determine the maximal height of the truck, such that the delivery is possible and provide a feasible route.



2 Graph coloring(/)

A vertex-coloring problem consists in assigning a legal color on each vertex of a graph, such that, any two adjacent vertices (connected by an edge) are assigned two different colors. For a graph G, the minimal number of colors respecting above constraint is called chromatic number of graph G, denoted $\chi(G)$.

Questions :

1. Draw the undirected graph with eight vertices (A1 to A8), represented here by the edges

   (A1,A2), (A1,A3), (A1,A4), (A1,A7)
   (A2,A3), (A2,A4), (A2,A5)
   (A3,A5), (A3,A6), (A3,A7)
   (A4,A5), (A4,A7)
   (A5,A6), (A5,A7), (A5,A8)
   (A6,A8)
   (A7,A8)

2. The Welch-Powell algorithm gives an approximate solution in determining the chromatic number. Apply this algorithm on the above graph and present the result of obtained number of colors.

   Welch-Powell algorithm
   - Step 1 : Order the vertices of the graph G by the decreasing order of vertices degrees.$i = 1$
   - Step 2 : Assign the color C_i at the first not colored vertex, then in sequential order, assign color C_i to each vertex which is not adjacent with vertices already colored with C_i. $i < -i + 1$
   - Step 3 : Repeat the step 2 until all the vertices are colored.

3. Explain why above graph is not 2-colorable (ie using only two colors) ? Determine the value of $\chi(G)$ for this graph.

4. Can you determine the chromatic number of $\chi(K_n)$, where K_n is a n vertices complete graph ?

5. Give the time complexity (using O-notation) of Welch-Powell algorithm based on n (number of vertices) and m (number of edges).

6. Theorem : the three following propositions are equivalent
   - G is 2-colorable.
   - G is a bipartite graph.
   - Every cycle in G contains an even number of edges.

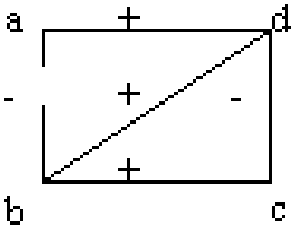
   Give the proof of this theorem (equivalence of propositions)

7. The following Table summarizes the candidates selection examination session, in which the lines correspond to the various examination subjects and columns to the candidates. If each candidate is authorized to participate only one exam per day, how many days are required to organize an examination session that all candidates can finish all the examination subjects ? Formulate the problem as a vertex-coloring problem. Specify the notation for the vertices and edges in the graph, the meaning of colors. Find a solution for such problem.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1				*		*	*		*	*			
2	*					*				*	*		
3		*						*			*		*
4	*			*									
5			*		*					*			*
6			*					*					
7	*	*							*		*		
8			*				*					*	

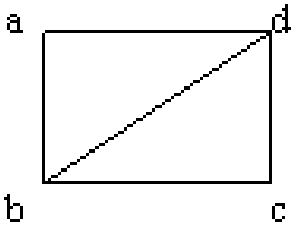
3 Signed graph (/)

roups dynamics is an interesting and important problem in social psychology. A group of people and their relationships can be represented by a graph, in which the vertices represent the individuals and the edges represent the relations among them. If there is an edge conjoining two vertices, that indicates there is a relationship between these two individuals. The problem of balance in Group dynamics was introduced by the psychologist Heider, who represents the problem by a signed graph. A signed graph is a graph $G = (S, A)$, in which a positive (+) or negative (-) sign is assigned on each edge which indicates the agreement or disagreement between two adjacent individuals. A path or a cycle in a signed graph is called positive if it has an even number of negative edges, and the one containing an odd number negative edges is called negative. A signed graph G is said to be balanced if and only if every cycle of G is positive.
Example :

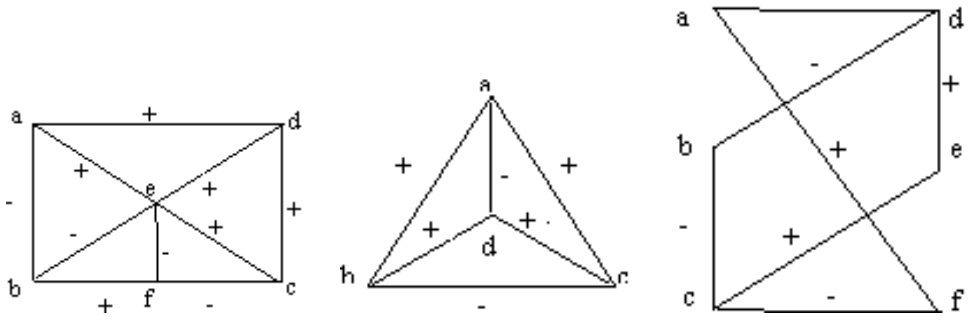


(a,d) ; (b,d) ; (b,c) are three pairs of individuals in agreement
a and b are in disagreement, c and d are in disagreement
the relationship between a and c is not known
The path adbc is positive , the path adbcd is negative The cycle adba is negative, the cycle adcba is positive.

1. Give (without considering isomorphic graphs) all complete signed graphs with three vertices. Indicate if it is balanced or not.



2. Add sign $+/-$ on the edges of above graph to obain three balanced graphs and three not balanced graphs.
3. Indicate the balanced graphs among the following graphs. Show for each balanced graph that its vertex set can be partitioned into two subsets (one of which may be empty) so that any edge joigning two vertices within the same subset is positive while any edge joigning two vertices in different subsets is negative.



The graph is said to be balanced if there exists a partition of the graph into two subsets (one of which may be empty) so that any edge joigning two vertices within the same subset is positive while any edge joigning two vertices in different subsets is negative. Prove the two theorems.

- (Theorem 1) A signed graph G is balanced if and only if all paths joining the same pair of vertices have the same sign
- (Theorem 2) A signed graph G is balanced if and only if every cycle of G is positive

4 Incompatibility (/)

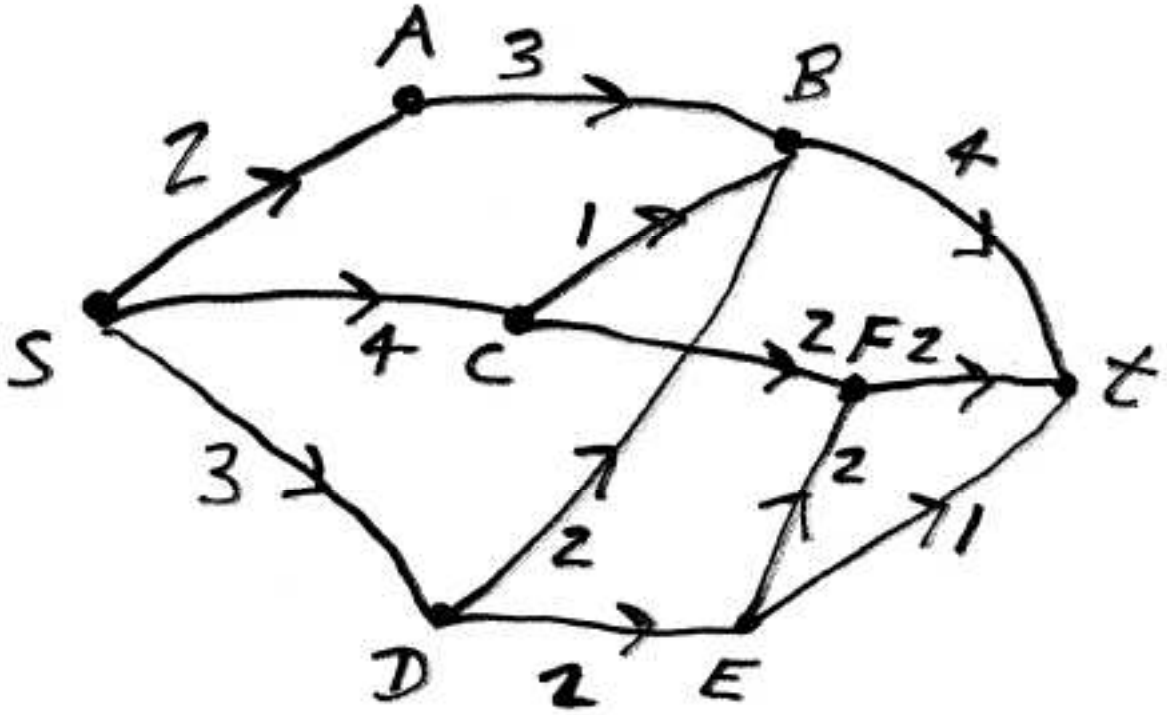
A group of eight people is meeting for dinner. The incompatible relations among the members are summarized in the following table :

	A	B	C	D	E	F	G	H
does not agree with	B,D	A,F,E,H	D,E	A,C,G	B,C,F	B,E,H	H,D	B,F,G

1. Model the conflicts among people in the form of a graph (specify the notation of vertices and edges).
2. Propose a plan table (the table is circular) for this group by separating the any two incompatible persons. Show that the problem consists in finding a particular set of vertices in a particular graph (specify which set in which graph).

5 Flow

Consider the directed network below. The numbers on the edges represent the capacity of each edge. Quote and apply an algorithm to find a maximum flow in this network, from s to t (Show at least one step of the algorithm). Also, find a minimum cut in the network.


A
3
B
2
4
1
S
4
C
2
F
2
t
3
2
2
1
D
2
E