U.T.B.M
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AG44

Final Exam - 2h

1 Tree

1. Show that from a connected graph G, it is always possible (by removing edges) to generate a subgraph $G' \subseteq G$ which is a tree.
2. Le G=(X,E) a connected graph which has exactly p connected components. Let n be the number of vertices. It wants to find a subgraph G'of G such that G' contains no cycle and has the maximum number of edges for this property. How many edges will have G'?

2 Friendly meeting

Suppose a group of friends from UTBM, wishes to organize a school reunion. They decide to find among them a contact who is easily reachable by route. The rule is to choose the one who is closest to all the others in terms of maximum distance (in term of graph theory, it is named eccentricity minimum). Since we want to take into account various factors (average accidents, probability of bad weather,...), a weights system is used to signify the difficulty on each road. In Figure 1, the nodes are the friends and the arcs between them show the accessibility between each two friends.

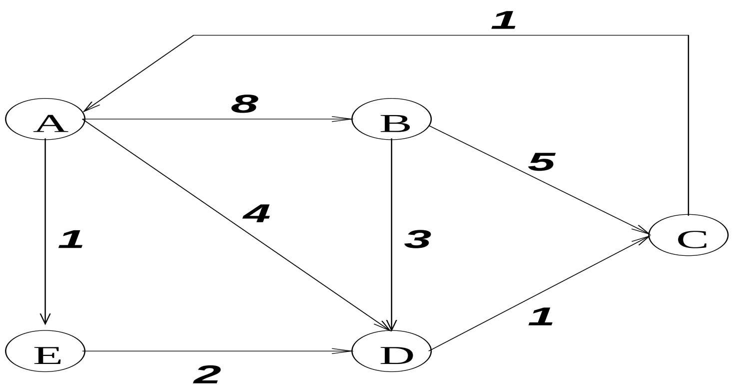


Figure 1 – A weighted and directed graph

We define the minimum eccentricity ecc of a person x in a directed weighted graph $G = (V, E)$. V denotes the set of vertices, E denotes the set of edges.

$$ecc(x) = \max_{y \in S}\{distance(y, x)\}$$

where $distance(y, x)$ denotes the minimum distance from vertex y to vertex x.

The selected contact will be in the "middle" to the others, who has the minimum eccentricity.

1. Write down an algorithm which allows to compute the minimum eccentricity of the graph
2. Find the contact in Figure 1, details your iteration of the algorithm to find this person.
3. Give the complexity of your algorithm in the number of vertices n and/or in the number of edges m .

3 Gallery

The director of the exhibition wants to make sure that the visit indications ("Direction of the visit") are properly positioned, and the doors in the trip are open. He constructs the trips as following table which indicates the direct next halls of each hall :

hall	next hall
1	$2, 3$
2	3
3	4
4	$2, 6$
5	$-$
6	$5, 7$
7	5

Except the doors between halls and their direct next halls, there are also the doors between hall 1 and 4, hall 4 and 5, hall 5 and 6, hall 6 and 7.

1. What is the minimum number of doors that the director must open, so that we can move from any hall to any other hall ? Why and which ones ?
2. In which direction should the director authorize the passage ? For your answer, draw the corresponding graph and give your explanation.

4 Vital arc

We define a most vital arc of a network as an arc whose deletion causes the largest decrease in the maximum s-t-flow value. Let f be an arbitrary maximum s-t-flow. Either prove the following claims or show through counterexamples that they are false :

- (a) A most vital arc is an arc e with the maximum value of c(e).
- (b) A most vital arc is an arc e with the maximum value of f (e).
- (c) A most vital arc is an arc e with the maximum value of f (e) among arcs belonging to some minimum cut.
- (d) An arc that does not belong to some minimum cut cannot be a most vital arc.
- (e) A network might contain several most vital arcs.

5 Maximal height

The local travelling agent wants to organize a trip in the Alps region, which determines the route connection among a set of places. The only concern is to avoid the high cols. The Figure 2 gives detail information of altitude difference between each two places.

1. Knowing all the places and the maximum altitude between each pair of neighborhood places, formulate this problem as a graph problem.
2. In Figure 2, let the weights on the adjacent places be the maximum altitude difference between two places. Please find a trip avoid the high altitude difference, in case that the trip will visiting all places on the map.

6 Dining problem

Several families go out for dinner together. In order to increase their social relation, they decide to organize the table arrangement as follows : there is no two members of same family sitting around same table. Formulate the table arrangement which satisfies such objective as a maximum flow problem. Assume there are p families and ith family has $a(i)$ members. Also assume that q tables are available and jth table has $b(j)$ places.

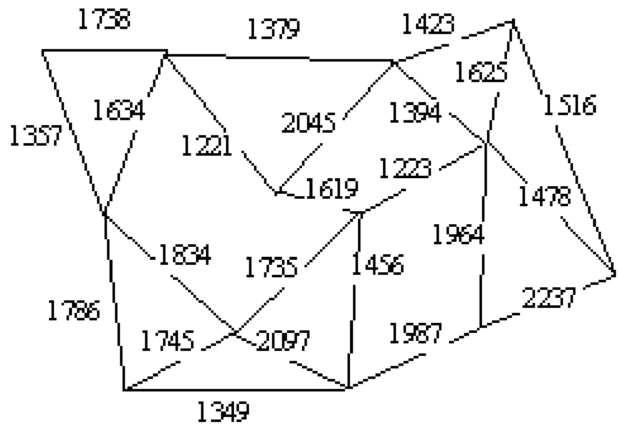
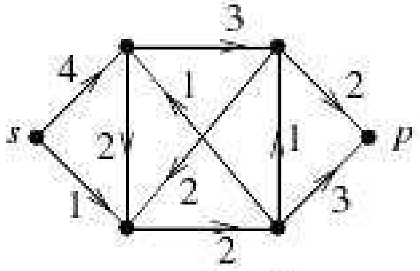


Figure 2 – Graph with different places of the trip

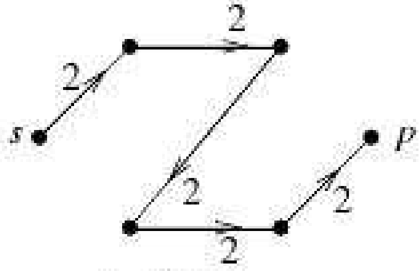
7 Flow

Let N be the flow network and f_0 the (s, p)-flow in N as depicted in the figure below.

1. Start from f_0 and find a maximum (s, p)-flow. Detail the steps of the algorithm.
2. Determine a minimum cut of this network.



network



initial flow