U.T.B.M
K. Deschinkel

2014-2015
AG44

Final Exam - 2h

1 New definitions /3

Let G = (V ;E) be a connected graph and $v \in V$. Let us introduce the following concepts :

- The eccentricity of the vertex v, $e(v)$, is the maximum of the distances from v to any other vertex of the graph, that is, $e(v) = max\{d(v, x) : x \in V\}$.
- The radius of G, $r(G)$, is the minimum of the eccentricities of the vertices of G, that is, $r(G) = min\{e(v) : v \in V\}$.
- The diameter of G, $D(G)$, is the maximum of the eccentricities of the vertices of G, that is, $D(G) = max\{e(v) : v \in V\}$.
- A central vertex of G is a vertex u such that $e(u) = r(G)$.

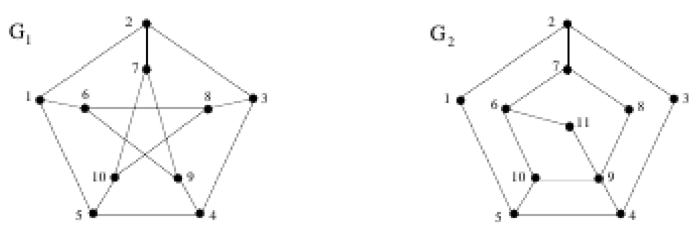

Figure 1 – Graphs G1 and G2

1. Find the eccentricities, the radius and the central vertices of the graph on figure 2.
2. Give an example of a graph with the same radius and diameter.
3. Give an example of a graph whose diameter is twice its radius.

PS : The **distance** between two vertices in a graph is the number of edges in a shortest path (also called a graph geodesic) connecting them.

2 Shortest Path /4

Tourists are staying in a hotel named A. A guide suggests a tour of six attractions named B, C, D, E, F and G. The road sections and the lenght (in kilometers) between two attractions are specified on the following graph :

1. From the hotel, is it possible to find a visit that goes through each road section exactly once ?
2. Same question if the guide suggest a circuit starting and finishing at the hotel and that goes through each road section exactly once.
3. Determine the shortest path from the hotel to the attraction E with the proper algorithm, justifying your choice.

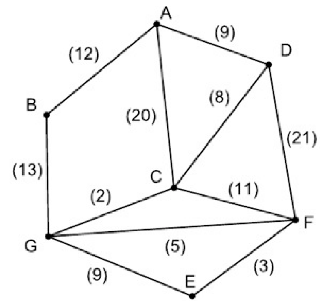

Figure 2 – Graph for tourist tour

3 Minimum spanning Tree /3

A graph G has edges which are either dashed or not.

1. Give the fastest algorithm that you can to compute a spanning tree with as few dashed edges as possible.
2. Apply your algorithm on figure 3

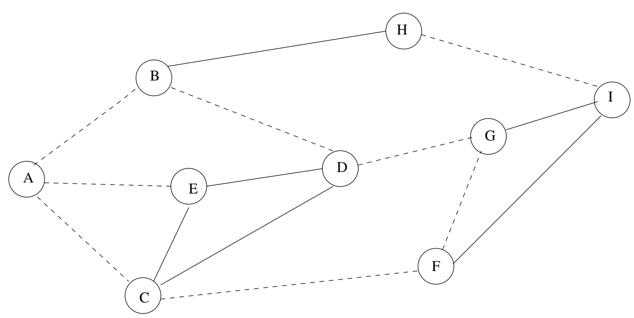

Figure 3 – Dashed graph

4 Flow /4

Consider the following network on figure 4. Capacities are given in bracket and flows in parentheses. We consider the flow of value 10.

1. Prove by using Ford-Fulkerson algorithm that the given flow is optimal.
2. We try to increase the flow value by increasing the arc capacities. Determine a set of three arcs, called A, satisfying the following property : to obtain a flow with a value greater than 10, you have to increase the capacity of at least one arc in A. Justify your answer.
3. Select among the arcs of A, one whose capacity is minimal and show that the increase of its capacity will necessary increase the flow value.
4. Increase of one unit the capacity of the arc selected in previous question and compute , the new maximal flow value.

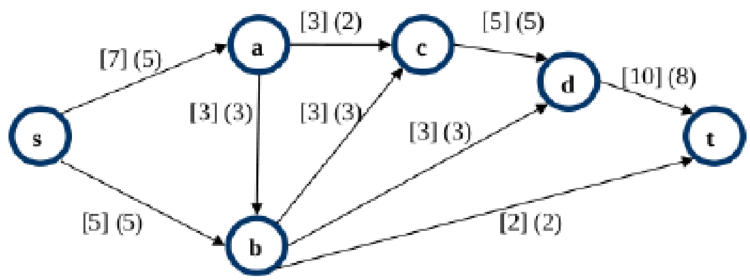

Figure 4 – Graph with flow[capacity] on each arc

5 Graph model /3

The city council of a city (composed of councilmans) includes seven committee, which obey the following rules :

- Rule 1 : every councilman is part of exactly two committes.
- Rule 2 : any two committees have exactly one councilman in common

1. Model the problem using a graph (specify vertices and links)
2. How many councilmans are in the city council ?
3. Deduct the number of councilmans in each committee.

6 Graph traversal /3

A mixed graph is graph in which some of the edges are directed and some of the edges are undirected. If a given mixed graph G has no directed cycle, then it is always possible to orient the remaining undirected edges so that the resulting graph has no directed cycle.

1. Give an efficient algorithm for obtaining such an orientation if one exists. Prove the correctness of your algorithm.
2. Apply your algorithm on figure 5

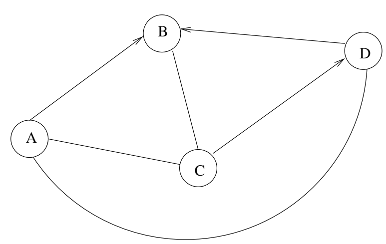

Figure 5 – Mixed graph