U.T.B.M
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AG44

Final Exam - 2h

1 Maximal Flows

Part 1

Enthusiastic celebration of a sunny day at a prominent northeastern university has resulted in the arrival at the university's medial clinic of 169 students in need of emergency treatment. Each of the 169 students requires a transfusion of one unit of whole blood. The clinic has supplies of 170 units of whole blood. The number of units of blood available in each of the four major blood groups and the distribution of patients among the groups is summarized below.

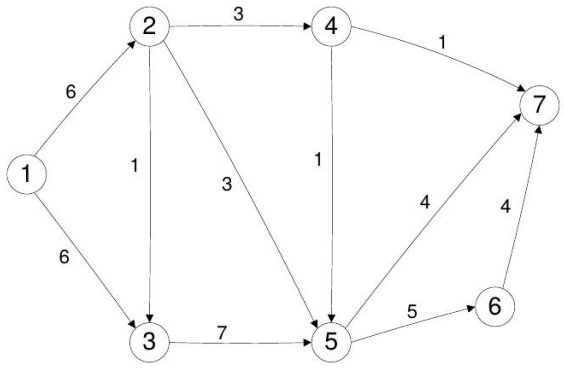
Blood type	A	B	O	AB
Supply	46	34	45	45
Demand	39	38	42	50

Type A patients can only receive type A or O; type B patients can receive only type B or O; type O patients can receive only type O; and type AB patients can receive any of the four types.

— Give a max flow formulation that determines a distribution that satisfies the demands of a maximum number of patients. Draw the directed graph and put the edge capacity above each edge. Your network should have 10 vertices: a source (named 0), a supply node for each of the four blood types (named 1 to 4), a demand node for each blood type (named 5 to 8), and a sink (named 9).

Part 2

Given the following network with capacities on the arcs, find a maximum (feasible) flow from node 1 to node 7, and determine a corresponding minimum (capacity) cut.



2 Structure

Is the degree sequence 4,3,3,2,2,1,1 graphical? Explain why, and produce, if possible, the corresponding graph.

3 Modelisation

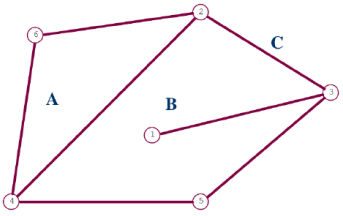
You possess a bank-note of p euros and you want to change it to coins of $a_1, a_2, ..., a_n$ euros. Is it possible? If yes, what is the minimum number of coins? Formulate this problem as a shortest path problem and solve it using the Dijsktra's algorithm (when $p = 8$, and three types of coins $a_1 = 1$, $a_2 = 3$ and $a_3 = 5$ euros)

4 Coloration

1. What is the chromatic number of the complete graph K_n?
2. What is the chromatic number of the path P_n?
3. What is the chromatic number of the cycle C_n?
4. Prove or disprove and justify :Every k-chromatic graph G has a proper k-colouring in which some colour class has $\alpha(G)$ (independence number of G) vertices.

5 Planar graph

In the graph below, give the number of faces and the degree of each face. Give the number of edges whithout counting them (give a formula to deduce this number)? Check the Euler's formula between the number of vertices, edges and faces for this graph.



6 Minimum spanning Tree

Part 1
Can you think of an algorithm for finding a spanning tree of a connected undirected graph?
Part 2
Can you think of a way to bound the solution to a TSP (traveling salesman problem) problem on an undirected connected graph using minimum spanning trees.
Part 3

1. Run Prim's algorithm; whenever there is a choice of nodes, always use alphabetic ordering (e.g.,start from node A). Draw a table showing the intermediate values of the cost array. Show the final spanning tree.
2. Suppose Kruskal's algorithm is run on this graph. In what order are the edges added to the MST? Show the final spanning tree.