U.T.B.M
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AG44

Median Exam - 2h

1 Clique /5

Let G be a graph with vertex set V. Let V1 be a subset of V. If the graph induced by V1 is a complete graph then we say that V1 is a clique in G. We shall often just call V1 a "clique". If a clique is not contained in any other clique in G then we say it is a maximal clique. A clique whose size is greater than or equal to that of every other clique in G is called a maximum clique. For a graph G, the size of a maximum clique in G is called the clique number of G.

1. Prove that if a graph G has a maximum clique with only one vertex then G is a null graph.
2. Prove that a graph G has exactly one maximal clique if and only if G is a complete graph.
3. Prove an inequality that relates the number of maximal cliques m to the number of connected components n in a graph. Give an example of a graph where m = n.
4. Your friend claims that if a graph has maximum cliques of size 2 then the number of maximum cliques is one half of the sum of the degrees of the vertices. If you believe the statement prove that it is true. If not, prove that it is not true.
5. How many maximum cliques are there in the complete bipartite graph $K_{r,s}$? Why ?
6. How many graphs of n vertices have a clique number equal to $n-1$? Explain why ?

2 Vertex connectivity /5

A vertex v in a connected graph is an articulation point if the removal of v and all edges with v as an end-point from the graph would leave a disconnected graph. A set of vertices whose removal would disconnect the graph is called a vertex cut. Typically a graph will have many vertex cuts. The vertex connectivity of a graph is the size of its smallest vertex cut(s). Often the vertex cut that achieves this minimum is not unique. If the vertex connectivity of a graph of a graph is k, then we say the graph is k-vertex connected.

1. How many articulation points does a graph circuit have ?
2. Prove that if a connected graph G with more than two vertices has no articulation points then for each pair of vertices w and v there are at least two distinct paths (with no vertices in common other than the end points) from w to v.
3. Draw a graph with exactly one vertex cut.
4. What is the vertex connectivity of the complete bipartite graph $K_{r,s}$? Why ?
5. If the vertex of lowest degree in a graph has degree d why must its vertex connectivity be less than or equal to d ? Draw an example where the vertex connectivity is less than d.

3 Bipartite graph /1

A bipartite graph is used in a certain college to model the relationship between students and courses. Let the vertices in one part, (call it V1) represent the students and the vertices in the other part (call it V2) represent the courses. What do the following represent ?

- the degree of a vertex s in V1
- the degree of a vertex c in V2
- the fact that two vertices s1 and s2 in V1 are adjacent to the same vertex c in V2.

4 Isomorphic graphs /1

Draw two isomorphic simple graphs $G = (V, E)$ and $G' = (V', E')$ with the degree sequence (1,2,2,2,3) and give the two functions $f : V \to V'$ and $g : E \to E'$ such that f (g) associates each element in V (E) with exactly one element in V' (E') and vice versa.

5 Degree sequence, vertices and edges /2

- Is it possible to construct a simple graph with 7 vertices and 23 edges ? If yes, draw the corresponding graph. If no, explain why.
- Is there a simple graph of 5 vertices with degrees 0,1,2,3,4 ? If yes, draw the corresponding graph.If no, explain why.
- Is there a simple graph of 5 edges having the degrees of its vertices 1,1,3,3 ? If yes, draw the corresponding graph. If no, explain why.
- Is the degree sequence 4,3,3,2,2,1,1 graphical ? Explain why, and produce, if possible, the corresponding graph.

6 Adjacency, incidence matrix and ... /3

Let G be the following directed graph :$G = (V, E)$ with :
- $V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}, \{(v_0, v_4), (v_0, v_1), (v_1, v_3), (v_2, v_0), (v_3, v_0), (v_3, v_5), (v_3, v_6), (v_6, v_1)\}$
- $E = \{\}$

1. Draw a representation of the graph G
2. Write the adjacency matrix of G
3. Write the incidence matrix of G
4. Give the minimum and maximum degree of its vertices
5. Give its edge-connectivity, its vertex-connectivity with a corresponding edge cut and vertex cut
6. Is the graph $H = (\{v_0, v_1, v_2, v_3, v_4\}, \{(v_0, v_4), (v_0, v_1), (v_1, v_3), (v_2, v_0)\}$ an induced subgraph of G ? Why ?
7. Give an example for a walk (between v_6 and v_0), a trail (between v_6 and v_0), a path (between v_6 and v_0), and a cycle in G

7 Graph traversal /3

Give the pre ordering and the post ordering obtained with a Depth First Search for graph G (starting at vertex v_3 and considering the vertices in the order of their number). Is it possible to perform topological sort ? If yes, write down it.