U.T.B.M
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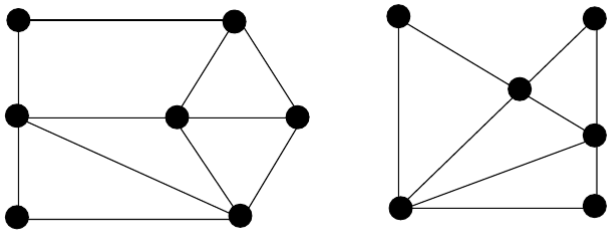
2017-2018
AG44

Median Exam - 2h

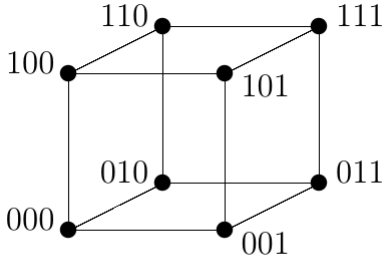
1 Euler (/)

1. Suppose a connected graph G has degree sequence $d_1, d_2, ..., d_n$. How many edges must be added to G so that the resulting graph has an Euler circuit? Explain.
2. Which complete graphs K_n, $n \geq 2$, have Euler circuits? Which have Euler trail? Justify your answers.
3. For each of the graphs below, state whether it has an Euler circuit. If the answer is yes, draw such a circuit (use arrows and number the sequence of edges in the order traveled).



2 Cube graph (/)

The cube graph Q_n is defined as follows: the vertices of Q_n are all sequences of lenght n with entries from $\{0, 1\}$ and two sequences are joined by an edge if they differ in exactly one position.



The cube graph Q_3

1. How many edges does Q_n have?
2. What is the degree sequence of Q_n?
3. Which cube graphs Q_n is eulerian?
4. Show that the cube graph Q_n is bipartite?

3 Incompatibility (/)

A group of eight people is meeting for dinner. The incompatible relations among the members are summarized in the following table :

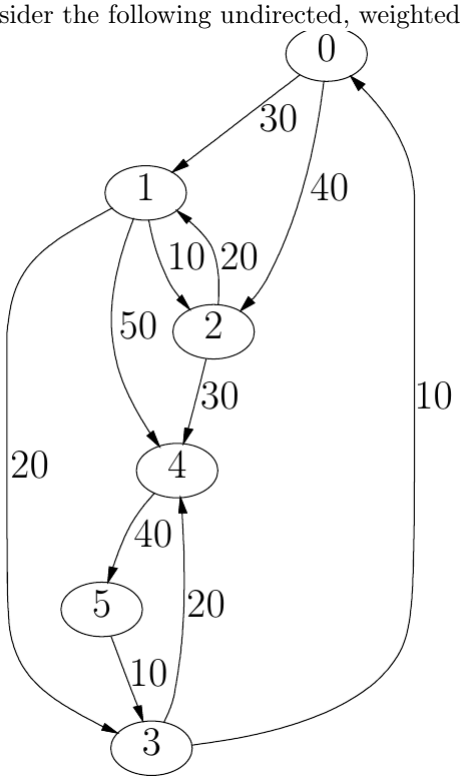
	A	B	C	D	E	F	G	H
does not agree with	B,D	A,F,E,H	D,E	A,C,G	B,C,F	B,E,H	H,D	B,F,G

1. Model the conflicts among people in the form of a graph (specify the notation of vertices and edges).
2. Propose a plan table (the table is circular) for this group by separating the any two incompatible persons. Show that the problem consists in finding a particular set of vertices in a particular graph (specify which set in which graph).

4 Shortest Paths (/)

4.1 Part 1

Consider the following undirected, weighted graph



Step through Dijkstra's algorithm to calculate the single-source shortest paths from 0 to every other vertex.

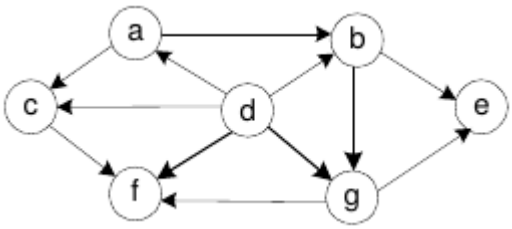
4.2 Part 2

Why the Dijkstra algorithm doesn't work with negative edges, could you give an example?

5 Graph Traversal (/)

5.1 Part 1

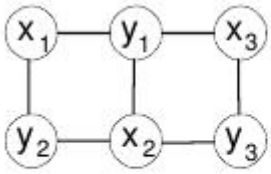
Apply the DFS-based algorithm to solve the topological sorting problem for the following digraph



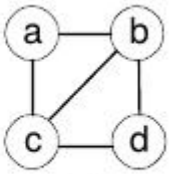
(for DFS, use the alphabetical order).

5.2 Part 2

1. A graph is said to be bipartite if its vertices can be colored in two colors so that every edge has its vertices colored in different colored in different colors. Design a DFS-based algorithm for cheking whether a graph is bipartite.
2. Apply this algorithm to say if the following graphs are bipartite or not.



(i)



(ii)

6 Vertex connectivity

Calculate the vertex and edge connectivity of the following graph. Thereafter, chose a vertex and delete it. Calculate the new edge and vertex connectivity. Does the choice of vertex matter?