AT54 EXAMEN FINAL

16 janvier 2012 de 8h00 à 10h00 en salle P131 à Sévenans

You can answer either in English or in French
No documents or PC's allowed.
Mobile phones are NOT ALLOWED IN THE CLASSROOM
Write clearly and avoid erasures
You have to solve all exercises to get a score of 20

PART1: SIGNAL PROCESSING

EXERCISE 1 (3 POINTS)

(a) Find the frequency response $H(e^{j\omega})$ of the linear time-invariant system whose input and output satisfy the difference equation:

$$y(n)-\frac{1}{2}y(n-1)=x(n)+2x(n-1)+x(n-2)$$

(b) Write a difference equation that characterises a system whose frequency response is

$$H(e^{j\omega})=\frac{1-\frac{1}{2}e^{-j\omega}+e^{-j3\omega}}{1+\frac{1}{2}e^{-j\omega}+\frac{3}{4}e^{-j2\omega}}$$

ATTENTION: use only the Fourier Transform

EXERCISE 2 (3 POINTS)

Consider a stable LTI system for which the z-transform of the impulse response is given by

$$H(z)=\frac{3}{1+\frac{1}{3}z^{-1}}$$

If the input x(n) of the system is a unit step sequence, compute y(n), the output, by using both the discrete convolution and the z-transform, and verify that the results are the same.

PART2: SYSTEM IDENTIFICATION

EXERCISE 3 (3 POINTS)

Consider the noise model below:

$$(1-a_1q^{-1}-a_2q^{-2})v(t)=(1-b_1q^{-1})e(t)$$

Where e(t) is white noise whose variance is λ

a) What kind of system is it?
b) What is the minimum variance estimator $\hat{v}(t|t-1)$?
c) What is the expected value and the variance of the estimation error ε?

EXERCISE 4 (4 POINTS)

Compare and describe the ARX and the ARMAX model structures; in particular find their minimum variance estimators. To what model family do they belong? Which is the other family of model structures studied during lectures, and what is its general form? In what respect do these two families mainly differ?

EXERCISE 5 (4 POINTS)

Given the linear overdetermined (what does that mean?) system $Ax \approx b$ *derive the least square solutions, by minimizing* $\varepsilon = \|b - Ax\|_2^2$ *, as explained in the lectures.*

EXERCISE 6 (3 POINTS)

Consider a Gaussian white noise with variance σ^2 *and null mean. Then if v(t)=H(q)e(t), define* $\hat{v}(t|t-1)$ *the minimum variance estimator*

1) *What is the MAP value of v(t) given the information* $\hat{v}(t|t-1)$*?*
2) *What id the probability that v(t) has a value between* $\hat{v}(t|t-1) - \infty$ *and* $\hat{v}(t|t-1) + \infty$*?*

Remember that a Gaussian function of mean μ and variance σ^2 *is given by:*

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

USEFUL DOCUMENTATION

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \quad (4.33)$$

TABLE 4.1 Some Common Black-box SISO Models as Special Cases of (4.33)

Polynomials Used in (4.33)	Name of Model Structure
B	FIR (finite impulse response)
AB	ARX
ABC	ARMAX
AC	ARMA
ABD	ARARX
ABCD	ARARMAX
BF	OE (output error)
BFCD	BJ (Box–Jenkins)

TABLE 3.1 SOME COMMON *z*-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$