# AT54 EXAMEN FINAL

27 juin 2010 de 10h15 à 12h15 en salle P227 à Sévenans

**You can answer either in English or in French**
**No documents or PC's allowed. Mobile phones must be TURNED OFF**
**Write clearly and avoid erasures**

## PART1: SIGNAL PROCESSING

**EXERCISE 1 (3 POINTS)**

*Consider the following LTI system, whose Z-transform of the impulse response is:*

$$H(z) = \frac{\left(1 - z^{-1}\right)}{\left(1 - 2z^{-1}\right)\left(1 - 5z^{-1}\right)} \qquad |z| < 2$$

*Compute :*

a) *h(n), that is the impulse response*
b) *The corresponding difference equation if x[n] is the input sequence and y[n)] the output sequence.*
c) *Is the system stable ? Is it causal ? Why?*
d) *Compute the output y(n) if a unit step function is given (considering null initial conditions). Make the computation by using BOTH the Z-transform AND the convolution product, and verify that the result is the same.*

   *(Hint:* $\sum_{k=-\infty}^{n} a^k = \frac{a^{n+1}}{a-1}$ *if* $|a| > 1 \ldots$*why ?)*

**EXERCISE 2 (3 POINTS) (Theory and practice)**

Define the DFT computed on N-points, by using the notion of DFS. Then consider the following finite-length sequence:

$$x[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0 & elsewhere \end{cases}$$

Compute the 5-point DFT of *x[n]*.

## PART2: SYSTEM IDENTIFICATION

**EXERCISE 3 (2 POINTS)**

*Consider the noise model below*

$$v(t) = (1 + a_1 q^{-1} + a_2 q^{-2})^{-1} e(t)$$

*where* $e(t)$ *is white noise whose variance is equal to λ.*

a) *What kind of system is it ?*
b) *What is the estimator* $\hat{v}(t|t-1)$ *?*
c) *What is the value and the variance of the estimation error ε?*

**EXERCISE 4 (2 POINTS) (Theory)**

*Describe and compare the model structures OE and ARX. Then compute for each of them the minimum variance estimator. Draw a block diagram for each of them to explain their difference. What is the name and the form of the generalized OE?*

**EXERCISE 5 (3 POINTS) (Theory)**

a) *Define an ARARMAX system (which is not an ARMAX !)*
b) *Find the minimum variance estimator*
c) *Find the parameter vector, the regressor vector and the corresponding equation*

**EXERCISE 6 (3 POINTS)**

*Consider the following white noise e(t) :*

| *e(t)* | *-22* | *-3* | *0* | *2* | *4* | *5* |
|---|---|---|---|---|---|---|
| *probability* | *1/12* | *1/12* | *1/4* | *1/4* | *1/12* | *1/4* |

a) *Verify that e(t) is zero mean and compute its variance*

*Then, if* $v(t) = H(q)e(t)$*, call* $\hat{v}(t|t-1)$ *the minimum variance estimator of v(t).*

b) *What is the MAP value of v(t) given the information* $\hat{v}(t|t-1)$ *?*
c) *What is the probability that v(t) has a value between* $\hat{v}(t|t-1)-1$ *and* $\hat{v}(t|t-1)+2$ *?*

**EXERCISE 7 (4 POINTS)**

*The following data pairs are given, which are the measured inputs and outputs of a system described by the function* $y(x) = \alpha_1 3^{\alpha_2 x}$ .

| $x_i$ | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| $y_i$ | $\sqrt{27}$ | *1* | *1* | *81* |

*Determine the parameters $\alpha_1$ and $\alpha_2$* so that the function *y(x) approximates the data* $(x_i, y_i)$ *in the least-squares sense. (Hint: make a suitable change of variables in order to use the least-squares method).*

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**USEFUL DOCUMENTATION**

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \quad (4.33)$$

**TABLE 4.1 Some Common Black-box SISO Models as Special Cases of (4.33)**

| Polynomials Used in (4.33) | Name of Model Structure |
|---|---|
| *B* | FIR (finite impulse response) |
| *AB* | ARX |
| *ABC* | ARMAX |
| *AC* | ARMA |
| *ABD* | ARARX |
| *ABCD* | ARARMAX |
| *BF* | OE (output error) |
| *BFCD* | BJ (Box–Jenkins) |

**TABLE 3.1 SOME COMMON $z$-TRANSFORM PAIRS**

| Sequence | Transform | ROC |
|---|---|---|
| 1. $\delta[n]$ | $1$ | All $z$ |
| 2. $u[n]$ | $\frac{1}{1-z^{-1}}$ | $\|z\| > 1$ |
| 3. $-u[-n-1]$ | $\frac{1}{1-z^{-1}}$ | $\|z\| < 1$ |
| 4. $\delta[n-m]$ | $z^{-m}$ | All $z$ except 0 (if $m > 0$) or $\infty$ (if $m < 0$) |
| 5. $a^n u[n]$ | $\frac{1}{1-az^{-1}}$ | $\|z\| > \|a\|$ |
| 6. $-a^n u[-n-1]$ | $\frac{1}{1-az^{-1}}$ | $\|z\| < \|a\|$ |
| 7. $na^n u[n]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\|z\| > \|a\|$ |
| 8. $-na^n u[-n-1]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\|z\| < \|a\|$ |