AT54 Final Exam

16th January 2014 from 8h00 to 10h00 in classroom P323 at Sevenans

No documents allowed. Mobile phones are to be turned off.

No I-phone, I-pad, Tablet, Portable PC, Calculator allowed.

1st part : SIGNAL PROCESSING

EXERCICE 1 (3 POINTS)

Consider the following LTI system,

$$H(z)=\frac{1}{(1-3z^{-1})(1+4z^{-1})} \qquad |z|>4$$

Compute:

a) *h(n) and its initial value.*
b) *The corresponding difference equation.*
c) *Is the system stable ? Is it causal ? Why ?*

EXERCISE 2 (3 POINTS)

Consider an LTI system that is stable and for which H(z), the z-transform of the impulse response, is given by

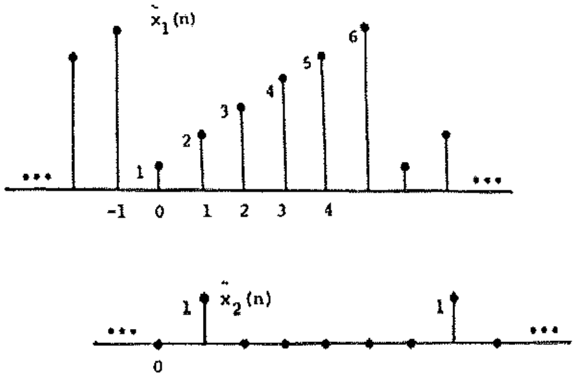
$$H(z)=\frac{3}{1+2z^{-1}}$$

Suppose x[n], the input of the system, is a unit step sequence.

a) *Find the output y[n] by evaluating the discrete convolution of x[n] and h[n]*
b) *Find the output y[n] by computing the inverse z-transform of Y(z).*

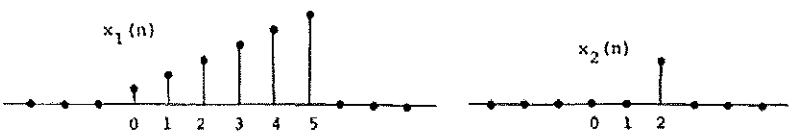
EXERCICE 3 (3 POINTS)

In the following figure two periodic sequences are shown both with period N=6. Compute graphically their periodic convolution of them $\tilde{x}_3[n]$



EXERCICE 4 (3 POINTS)

In the following figure two finite length sequences are sketched. Compute the 6-point circular convolution:



2^{nd} part : SYSTEM IDENTIFICATION

EXERCICE 5 (2 POINTS)

Let the noise be described by a LTI system as follows :

$$v(t) = \frac{(1 - a_1 q^{-1})}{(1 - b_1 q^{-1})} e(t)$$

where $e(t)$ *is a white noise signal whose variance is* λ.

a) *What kind of noise model is it ?*

b) *What is the mimimum variance predictor* $\hat{v}(t|t-1)$ *?*

c) *What is the value of the prediction error and its variance? Comment on your answer.*

EXERCICE 6 (3 POINTS) (Theory)

a) *Define the OE and the ARX systems*

b) *For both systems find* $y(t|t-1)$

c) *For both systems find the regressor , the parameter vector and the regression equation.*

d) *Can you use the linear least-squares method to compute their parameters? Why?*

EXERCICE 7 (3 POINTS) (Theory)

Let a real system LTI be described by the following model :

$y(t) = G(q)u(t) + H(q)e(t)$ *where* $e(t)$ *is the white noise*

Define and prove $y(t|t-1)$ *on the base of the knowledge of the past values of y(t) till the instant(t-1). To do this you need to define* $\hat{v}(t|t-1)$ *starting from the LTI noise model* $v(t) = H(q)e(t)$

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$