AT54 EXAMEN MEDIAN

8 November 2011 from 16h15 to 18h15 in classroom B429 in Belfort

No documents permitted.
One exercise of the theoretical part should be solved to validate the exercises of part 1.
No answer of the theoretical part may compromise the final score.

PART 1: EXERCISES

EXERCISE 1 (3 POINTS)

Determine the output of a LTI with impulse response h[n] and input x[n] by using the convolution, and not the transform methods, and by using your knowledge of linearity and time-invariance to minimize the work:

a) $x[n] = u[n]$ *and* $h[n] = a^n u[-n-1]$ *with a>1*
b) $x[n] = u[n-4]$ *and* $h[n] = 2^n u[-n-1]$

EXERCISE 2 (3 POINTS)

Consider an LTI system whose input x(n) and output y(n) satisfying the difference equation

$$y[n] + \frac{3}{2} y[n-1] + \frac{1}{2} y[n-2] = x[n] - 2x[n-1]$$

Determine all possible values for the system's impulse response h[n] and moreover h[0] for each of them.

EXERCISE 3 (4 POINTS)

Consider an LTI system that is stable and for which H(z), the z-transform of the impulse response, is given by

$$H(z) = \frac{2}{1+3z^{-1}}$$

Suppose x[n], the input of the system, is a unit step sequence.

a) *Find the output y[n] by evaluating the discrete convolution of x[n] and h[n]*
b) *Find the output y[n] by computing the inverse z-transform of Y(z).*

PART 2 : THEORETICAL QUESTIONS

EXERCISE 5 (3 POINTS)

Write the "Initial-Value Theorem" for a sequence x[n] whose Z-Transform is X(z) and then prove it. Make sure to well establish the assumption.

EXERCISE 6 (2 POINTS)

Prove that if a sequence is $x[n-n_0]$ *then its Z transform is* $z^{-n_0} X(z)$ *(of course X(z) is the z-transform of x[n]).*

EXERCISE 6 (5 POINTS)

Prove the following fundamental property of the convolution of sequences by using the Z-transform, where X(z) is the z-transform of the sequence x[n]:

$x_1[n] * x_2[n] \overset{Z}{\leftrightarrow} X_1(z)X_2(z)$ `with ROC the intersection of the ROC of each transform`

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n-n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\|a\| < 1)$	$\dfrac{1}{1-ae^{-j\omega}}$
5. $u[n]$	$\dfrac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n+1)a^n u[n] \quad (\|a\| < 1)$	$\dfrac{1}{(1-ae^{-j\omega})^2}$
7. $\dfrac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\|r\| < 1)$	$\dfrac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\dfrac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \|\omega\| < \omega_c, \\ 0, & \omega_c < \|\omega\| \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$