MEDIAN **Spring 2023**

Duration: **90** minutes

- It is advisable to take knowledge of the entire text before answering any question.

- Applicants must respect the used notation and specify in each case the question number.

- Most attention will be given to the clarity of writing, presentation, the diagram and the presence of measurement unit

Results will be put in frames

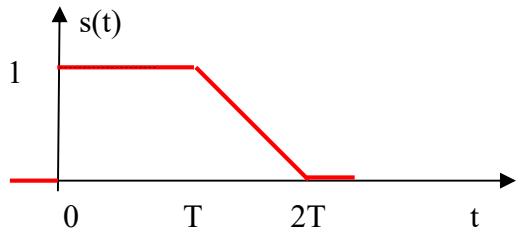
Exercises are independent

Documentation: An A4 double face is authorized, Calculator authorized, phone forbidden

The initial conditions are equal to zero for all exercises.

Exercise 1 (8pts):

Calculate the Laplace transform of the following signal (give all details of intermediate calculations):



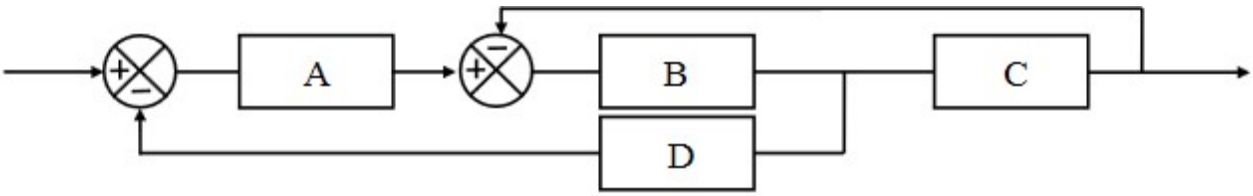
Exercise 2 (6pts):

Calculate the inverse Laplace transform of the following transfer function:

$$F(p) = \frac{p^2 + 5}{p(p^2 + 4)}$$

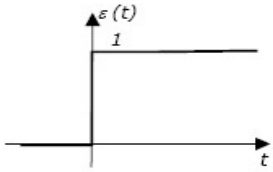
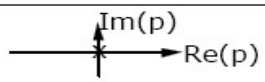
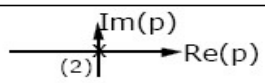
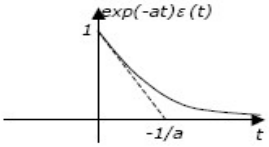
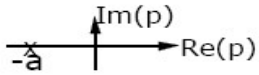
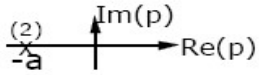
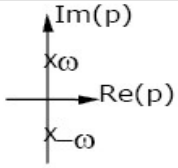
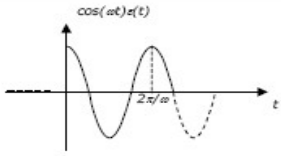
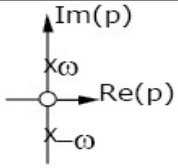
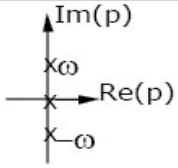
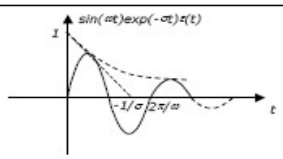
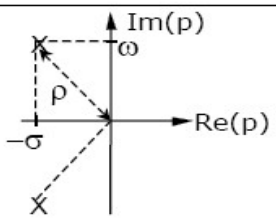
Exercise 3 (6pts):

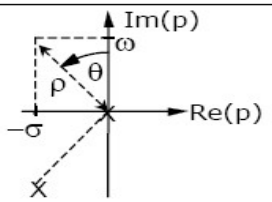
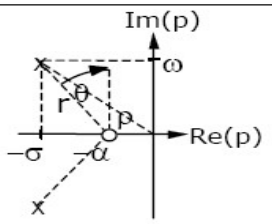
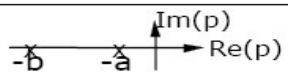
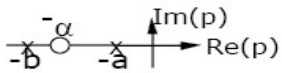
Simplify the following bloc scheme and give its closed loop transfer function H(p)



Annexes

Laplace Transforms

f(t)		F(p)	
$\delta(t)$		1	Ni pôle ni zéro
$\varepsilon(t)$		$\frac{1}{p}$	
$t\varepsilon(t)$		$\frac{1}{p^2}$	
$e^{-at}\varepsilon(t)$		$\frac{1}{p+a}$	
$te^{-at}\varepsilon(t)$		$\frac{1}{(p+a)^2}$	
$\frac{1}{\omega}\sin(\omega t)\,\varepsilon(t)$		$\frac{1}{p^2+\omega^2}$	
$\cos(\omega t)\,\varepsilon(t)$		$\frac{p}{p^2+\omega^2}$	
$\frac{1}{\omega^2}(1-\cos(\omega t))\,\varepsilon(t)$		$\frac{1}{p(p^2+\omega^2)}$	
$\frac{1}{\omega}e^{-\sigma t}\sin(\omega t)\varepsilon(t)$		$\frac{1}{p^2+2\sigma p+\rho^2}$ $(\sigma<\rho)$	

$\frac{1}{\rho^2}(1-e^{-\sigma t}\frac{\rho}{\omega}\cos(\omega t-\theta))$ $\theta=\arctan\left(\frac{\sigma}{\omega}\right)$	$\frac{1}{p(p^2+2\sigma p+\rho^2)}$ $(\sigma<\rho)$	
$\frac{r}{\omega}e^{-\sigma t}\cos(\omega t-\theta)\varepsilon(t)$ $\theta=\arctan\left(\frac{\alpha-\sigma}{\omega}\right)$ $r=\sqrt{\omega^2+(\alpha-\sigma)^2}$	$\frac{p+\alpha}{p^2+2\sigma p+\rho^2}$ $(\sigma<\rho)$	
$\frac{1}{b-a}(e^{-at}-e^{-bt})\varepsilon(t)$	$\frac{1}{p^2+2\sigma p+\rho^2}$ $(\sigma>\rho)$ $=\frac{1}{(p+a)(p+b)}$	
$(Ke^{-at}+(1-K)e^{-bt})\varepsilon(t)$ $K=\frac{\alpha-a}{b-a}$	$\frac{(p+\alpha)}{p^2+2\sigma p+\rho^2}$ $(\sigma>\rho)$ $=\frac{(p+\alpha)}{(p+a)(p+b)}$	

Properties

$L_I[af(t)+bg(t)]=aF(p)+bG(p)$ (linéarité)

$L_I\left[\frac{df(t)}{dt}\right]=pF(p)-f(0\_)$ (dérivée)

$L_I\left[\int_0^t f(t)dt\right]=\frac{F(p)}{p}$ (intégrale)

$L_I[f(t-\tau)]=e^{-p\tau}F(p)$ (retard temporel)

$L_I\left[e^{-\sigma t}f(t)\right]=F(p+\sigma)$ (translation de la transformée)

$L_I[f(t)*g(t)]=F(p)G(p)$ (convolution)

$\lim_{p\to\infty} pF(p)=\lim_{t\to 0+} f(t)$ (théorème de la valeur initiale)

(à condition que ces limites existent)

$\lim_{p\to 0} pF(p)=\lim_{t\to\infty} f(t)$ (théorème de la valeur finale)

(à condition que ces limites existent)

$L_I\left[\sum_{k=0}^{\infty} f(t-kT)\varepsilon(t-kT)\right]=\frac{F(p)}{1-e^{-pT}}$ (périodification)