

MEDIAN**Spring 2026****Duration: 90 minutes**

- It is advisable to take knowledge of the entire text before answering any question.
- Applicants must respect the used notation and specify in each case the question number.
- Most attention will be given to the clarity of writing, presentation, the diagram and the presence of measurement unit

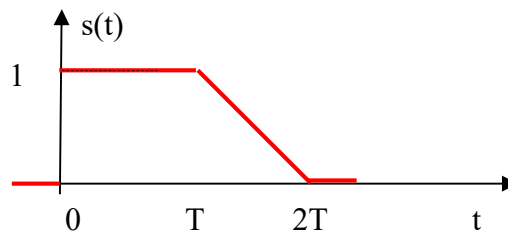
Results will be put in frames

Exercises are independent**Documentation: An A4 double face is authorized, Calculator authorized, phone forbidden**

The initial conditions are equal to zero for all exercises.

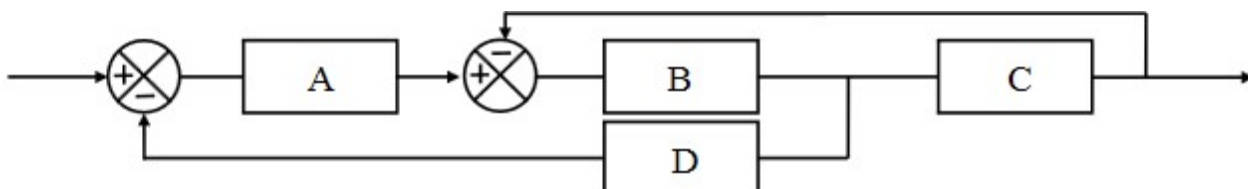
Exercise 1 (8pts):

Calculate the Laplace transform of the following signal (give all details of intermediate calculations):

**Exercise 2 (6pts):**

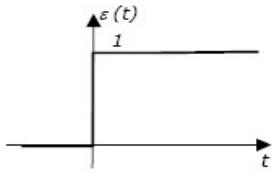
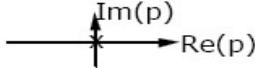
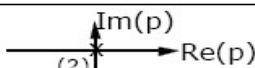
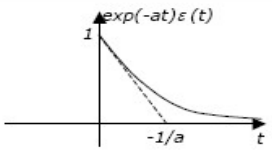
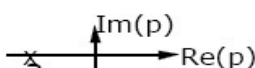
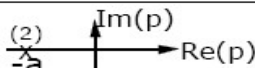
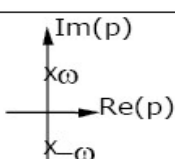
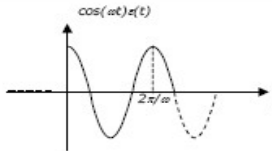
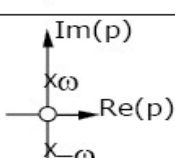
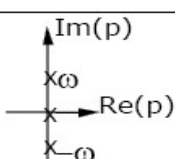
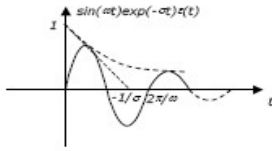
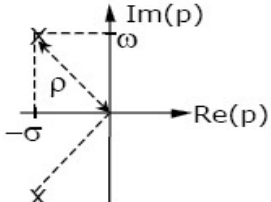
Calculate the inverse Laplace transform of the following transfer function:

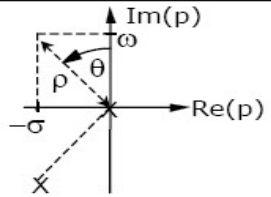
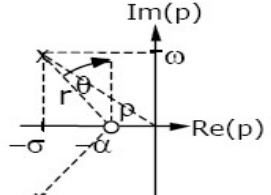
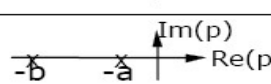
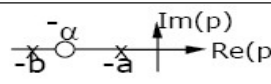
$$F(p) = \frac{p^2 + 5}{p(p^2 + 4)}$$

Exercise 3 (6pts):Simplify the following bloc scheme and give its closed loop transfer function $H(p)$ 

Annexes

Laplace Transforms

f(t)	F(p)
$\delta(t)$	1 Ni pôle ni zéro
$\varepsilon(t)$ 	$\frac{1}{p}$ 
$t\varepsilon(t)$	$\frac{1}{p^2}$ 
$e^{-at}\varepsilon(t)$ 	$\frac{1}{p+a}$ 
$te^{-at}\varepsilon(t)$	$\frac{1}{(p+a)^2}$ 
$\frac{1}{\omega}\sin(\omega t)\varepsilon(t)$	$\frac{1}{p^2 + \omega^2}$ 
$\cos(\omega t)\varepsilon(t)$ 	$\frac{p}{p^2 + \omega^2}$ 
$\frac{1}{\omega^2}(1 - \cos(\omega t))\varepsilon(t)$	$\frac{1}{p(p^2 + \omega^2)}$ 
$\frac{1}{\omega}e^{-\sigma t}\sin(\omega t)\varepsilon(t)$ 	$\frac{1}{p^2 + 2\sigma p + \rho^2}$ ($\sigma < \rho$) 

$\frac{1}{\rho^2} (1 - e^{-\sigma t} \frac{\rho}{\omega} \cos(\omega t - \theta))$ $\theta = \arctan\left(\frac{\sigma}{\omega}\right)$	$\frac{1}{p(p^2 + 2\sigma p + \rho^2)}$ $(\sigma < \rho)$ 
$\frac{r}{\omega} e^{-\sigma t} \cos(\omega t - \theta) \varepsilon(t)$ $\theta = \arctan\left(\frac{\alpha - \sigma}{\omega}\right)$ $r = \sqrt{\omega^2 + (\alpha - \sigma)^2}$	$\frac{p + \alpha}{p^2 + 2\sigma p + \rho^2}$ $(\sigma < \rho)$ 
$\frac{1}{b-a} (e^{-at} - e^{-bt}) \varepsilon(t)$	$\frac{1}{p^2 + 2\sigma p + \rho^2}$ $(\sigma > \rho)$ $= \frac{1}{(p+a)(p+b)}$ 
$(Ke^{-at} + (1-K)e^{-bt}) \varepsilon(t)$ $K = \frac{\alpha - a}{b - a}$	$\frac{(p + \alpha)}{p^2 + 2\sigma p + \rho^2}$ $(\sigma > \rho)$ $= \frac{(p + \alpha)}{(p+a)(p+b)}$ 

Properties

$L_I [af(t) + bg(t)] = aF(p) + bG(p) \quad (\text{linéarité})$
$L_I \left[\frac{df(t)}{dt} \right] = pF(p) - f(0_-) \quad (\text{dérivée})$
$L_I \left[\int_0^t f(t) dt \right] = \frac{F(p)}{p} \quad (\text{intégrale})$
$L_I [f(t - \tau)] = e^{-p\tau} F(p) \quad (\text{retard temporel})$
$L_I [e^{-\sigma t} f(t)] = F(p + \sigma) \quad (\text{translation de la transformée})$
$L_I [f(t) * g(t)] = F(p)G(p) \quad (\text{convolution})$
$\lim_{p \rightarrow \infty} pF(p) = \lim_{t \rightarrow 0+} f(t) \quad (\text{théorème de la valeur initiale})$ (à condition que ces limites existent)
$\lim_{p \rightarrow 0} pF(p) = \lim_{t \rightarrow \infty} f(t) \quad (\text{théorème de la valeur finale})$ (à condition que ces limites existent)
$L_I \left[\sum_{k=0}^{\infty} f(t - kT) \varepsilon(t - kT) \right] = \frac{F(p)}{1 - e^{-pT}} \quad (\text{périodification})$