Médian EL30 : 04/11/08 · corrigé

(1)

Ex 1 : (05 points)

1 - $i_g = 0 \quad (V_A - V_B = 0)$

$$V_A - V_B = (V_A - V_C) + (V_C - V_B) = -R_1 i_1 + R_2 i_2 \Rightarrow R_1 i_1 = R_2 i_2$$

$$= R_4 i_1 \cancel{+} R_3 i_2 = 0 \Rightarrow R_4 i_1 = R_3 i_2 \Rightarrow i_1 = \frac{R_3 i_2}{R_4}$$

$$R_1 \cdot \frac{R_3}{R_4} i_2 = R_2 i_2 \Rightarrow \boxed{R_1 R_3 = R_2 R_4}$$ (2)

2 - Pont équilibré ⇒ ajuster un courant. (1)

3 - $e = 0$

$$E_{th} = (V_A - V_B)_0 = -R_1 i_1 + R_2 i_2 \qquad i_2 = i_3 = \frac{e}{R_2 + R_3}$$

$$= R_4 i_2 + R_3 i_1 \qquad i_1 = i_4 = \frac{e}{R_1 + R_4}.$$

$$E_{th} = -R_1 \cdot \frac{e}{R_1 + R_4} + R_2 \frac{e}{R_2 + R_3} = e \left(\frac{R_2 (R_1 + R_4) - R_1 (R_2 + R_3)}{(R_1 + R_4)(R_2 + R_3)} \right)$$

$$= \frac{-10}{5} + \frac{2 \times 10}{5} = -2 + 4 = 2\,V.$$

$$R_{th} = R_1 \parallel R_4 + R_2 \parallel R_3 = \frac{1.4}{1+4} + \frac{2.3}{2+3} = \frac{4}{5} + \frac{6}{5} = \frac{10}{5} = 2\,\Omega$$

$$i_g = \frac{E_{th}}{R_{th} + g} = \frac{2}{2+2} = 0{,}5\,A$$ (2)

Ex2 : 05 pts

1 - $i = C\dfrac{dV_s}{dt}$. 0,5

2 - $V_e = Ri + V_s$ 0,5

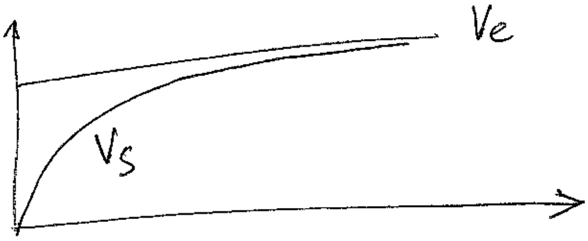
3 - $V_e = V_s + \tau\dfrac{dV_s}{dt}$ 0,5

4 - $V_e = E \quad t > 0 \quad V_s(t) = \lambda e^{-t/\tau} + E$

à $t=0 \quad V_s(0) = 0 \Rightarrow E = -\lambda$

$V_s(t) = E\left(1 - e^{-t/\tau}\right)$ 1

5 -



0,5

6 - $V_e(t) = at \quad t > 0$

on réinjecte l'équation proposée

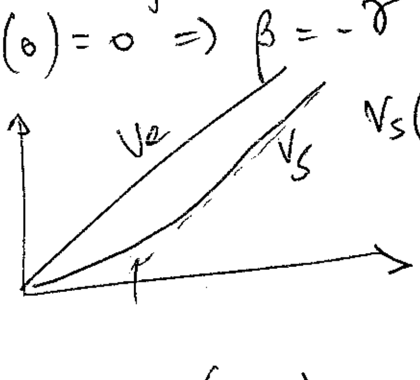
$at = \alpha t + \beta + \gamma \exp(-t/\tau) + \tau\left[\alpha - \dfrac{\gamma}{\tau}\exp(-t/\tau)\right]$ 1

$at = \alpha t + \beta + \alpha\tau$

D'où par identification $a = \alpha$ et $\beta = -\alpha\tau = -a\tau$

à $t=0 \quad V_s(0) = 0 \Rightarrow \beta = -\gamma \Rightarrow \gamma = a\tau$

7 -



$V_s(t) = a(t-\tau) + a\tau\exp(-t/\tau)$

0,5

8 - $t \gg \tau \Rightarrow \exp(-t/\tau) \to 0 \Rightarrow V_s = a(t-\tau)$

d'où décalage de τ de V_s/V_e. 0,5

Ex3 : 05 pts

1- $$E_{th} = \frac{Z_R \cdot e(t)}{Z_R + Z_L} = \frac{R\, e(t)}{R + \frac{1}{jC\omega}} = \frac{j\, e(t)}{1+j}$$

$$E_{th} = \frac{E_0\, e^{j(\omega t + \pi/2)}}{\sqrt{2}\, \exp j\pi/4} = \frac{E_0}{\sqrt{2}}\, e^{j(\omega t + \pi/4)}$$

1,5

$$Z_{th} = \frac{1}{jC\omega} + \frac{R}{1+jRC\omega} = \frac{R}{jRC\omega} + \frac{R}{1+jRC\omega} = \frac{R}{j} + \frac{R}{1+j}$$

$$Z_{th} = R\left(-j + \frac{1}{1+j}\right) = R\left(-j + \frac{1-j}{2}\right) = \frac{R}{2}\left(1 - 3j\right)$$

$$a = \frac{R}{2} \qquad b = -\frac{3R}{2}$$

1,5

$$|Z_{th}| = \sqrt{\left(\frac{R}{2}\right)^2 + \left(\frac{3R}{2}\right)^2} = \frac{R}{2}\sqrt{1+9} = \frac{R\sqrt{10}}{2}$$

$$\varphi = \arctan\left(\frac{-3}{1}\right) = -\arctan 3$$

2- $$\eta = \frac{E_{th}}{Z_{th}} = \frac{\frac{E_0}{\sqrt{2}}\, e^{j(\omega t + \pi/4)}}{\frac{R\sqrt{10}}{2}\, e^{+j\varphi}} = \frac{2E_0}{R\sqrt{20}}\, e^{j(\omega t + \pi/4 - \varphi)}$$

1

$$\eta = \frac{E_0}{R\sqrt{5}} \exp j\left(\omega t + \pi/4 - \varphi\right)$$

3- $$i_R = \frac{E_{th}}{Z_{th} + R} = \frac{E_{th}}{\frac{R}{2}(1-3j) + R} = \frac{E_{th}}{\frac{R}{2}\left(1 - 3j + 2\right)} = \frac{E_{th}}{\frac{R}{2}(3-3j)}$$

$$i_R = \frac{2E_{th}}{3R(1-j)} = \frac{2E_{th}}{3R\sqrt{2}\, e^{-j\pi/4}} = \frac{2E_0}{3R\sqrt{2}\sqrt{2}}\, e^{j(\omega t + \pi/4 + \pi/4)}$$

1

$$i_R = \frac{E_0}{3R}\, e^{j(\omega t + \pi/2)}$$

Ex4: 05 pts

4

1- $Z_{RL} = R + jL\omega$ 1

2- $Z_{RC} = R + \frac{1}{jC\omega}$ 1

3- $$Z_T = Z_{RL} \parallel Z_{RC} = \frac{(R+jL\omega)\left(R+\frac{1}{jC\omega}\right)}{(R+jL\omega)+\left(R+\frac{1}{jC\omega}\right)} = \frac{R^2 + jL\omega R + \frac{R}{jC\omega} + \frac{L}{C}}{2R + j\left(L\omega - \frac{1}{C\omega}\right)}$$

$$Z_T = \frac{\left(R^2+\frac{L}{C}\right) + jR\left(L\omega - \frac{1}{C\omega}\right)}{2R + jB}$$ 2

$A = R^2 + \frac{L}{C}$

$B = L\omega - \frac{1}{C\omega}$

Si $LC\omega^2 = 1 \Rightarrow B = 0$

$$Z_T = \frac{R^2 + L/C}{2R}$$

$LC\omega^2 = 1 \Rightarrow LC = \frac{1}{\omega^2}$

$C = \frac{1}{L\omega^2}$

$\frac{LC}{C^2} = \frac{1}{\omega^2 C^2}$

$$Z_T = \frac{R^2 + L^2\omega^2}{2R} = \frac{R}{2} + \frac{L^2\omega^2}{2R}$$

1