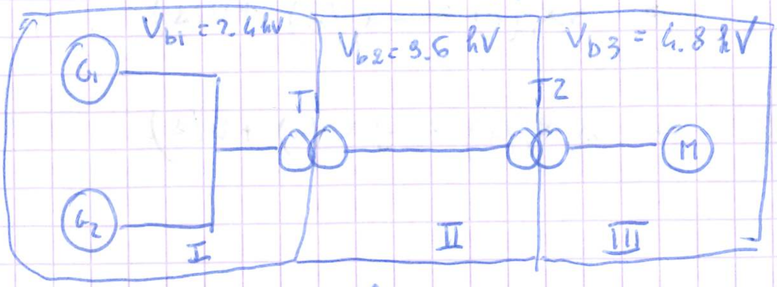
_ ER51 _ 25 June 2024

_ Exercise 1 _

$S_b = 100\ kVA = 0.1\ MVA$ $\quad V_b = 2400\ V = 2.4\ kV$ (in section I where the generators are)



$$Z_{new} = Z_{old} \left(\frac{S_{new}}{S_{old}}\right)\left(\frac{V_{old}}{V_{new}}\right)^2$$

G1 — 10 kVA, 2.4 kV, $z = j0.2$

$$Z_{G1} = j0.2\left(\frac{100}{10}\right)\left(\frac{2.4}{2.4}\right)^2 = j2\ pu$$

G2 — 20 kVA, 2.4 kV, $z = j0.2$

$$Z_{G2} = j0.2\left(\frac{100}{20}\right)\left(\frac{2.4}{2.4}\right)^2 = j1\ pu$$

T_1 — 40 kVA, 2.4/3.6, $z = j0.1$

$$Z_{T1} = j0.1\left(\frac{100}{40}\right)\left(\frac{2.4}{2.4}\right)^2 = 0.25$$

In zone III we have $\frac{V_{b2}}{V_{b3}} = \frac{10}{5} \Rightarrow V_{b3} = \frac{5}{10} V_{b2} = \frac{3.6}{2} = 4.8\ kV$

T_2 — 80 kVA, 10/5 kV, $z = j0.1$

$$Z_{T2} = j0.1\left(\frac{100}{80}\right)\left(\frac{10}{3.6}\right)^2 = j0.136\ pu$$

In zone II the base impedance is $Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{(3600)^2}{100 \cdot 10^3} = 921.6$

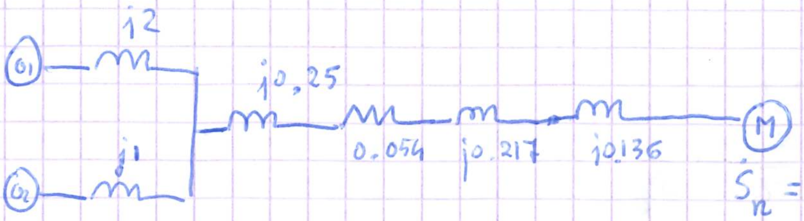
The line is $z = 50 + j200\ \Omega$ so in pu

$$\dot{z} = \frac{z}{z_{b2}} = \frac{50 + j200}{921.6} = 0.054 + j0.217\ pu$$

M: 25 kVA, 4 kV

Thus in p.u. $S_M = \frac{25\ kVA}{100} = 0.25$

$$V_n = \frac{4}{4.8} = 0.833\ pu$$



$\dot{S}_n = 0.25$ pu

$\dot{V}_n = 0.833$ pu

- Exercise 2 -

This exercise is explained in lecture 1, slide 31.

We have

$$L_1 = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{1}{R_1}\right) - \frac{\mu_0}{2\pi}\ln\frac{1}{D}$$

$$L_2 = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{1}{R_2}\right) - \frac{\mu_0}{2\pi}\ln\frac{1}{D}$$

Total inductance is therefore:

$$L = L_1 + L_2 = \frac{\mu_0}{2\pi}\left(\frac{2}{4} + \ln\frac{1}{R_1} + \ln\frac{1}{R_2}\right) - \frac{\mu_0}{2\pi}\ln\frac{1}{D} - \frac{\mu}{2\pi}\ln\frac{1}{D}$$

$$= \frac{\mu_0}{2\pi}\left(\frac{1}{2} + \ln\frac{1}{R_1R_2}\right) - \frac{\mu_0}{\pi}\ln\frac{1}{D} =$$

$$= \frac{\mu_0}{\pi}\left(\frac{1}{4} + \frac{1}{2}\ln\frac{1}{R_1R_2}\right) - \frac{\mu_0}{\pi}\ln\frac{1}{D} =$$

$$= \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{1}{\sqrt{R_1R_2}} - \ln\frac{1}{D}\right) =$$

$$= \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{D}{\sqrt{R_1R_2}}\right) \text{ H/m.}$$

Es n. 3

On P_1 on the surface of conductor 1

$$v_{P_1} = \frac{Q}{2\pi\varepsilon_0} \ln\frac{1}{R_1} - \frac{Q}{2\pi\varepsilon_0} \ln\frac{1}{D}$$

$$v_{P_2} = -\frac{Q}{2\pi\varepsilon_0} \ln\frac{1}{R_2} + \frac{Q}{2\pi\varepsilon_0} \ln\frac{1}{D}$$

$$\Delta v = v_{P_1} - v_{P_2} = \frac{Q}{2\pi\varepsilon_0}\left(\ln\frac{1}{R_1} - \ln\frac{1}{D} + \ln\frac{1}{R_2} - \ln\frac{1}{D}\right) =$$

$\ln\frac{1}{D} = -\ln D$

$$= \frac{Q}{2\pi\varepsilon_0}\left(\ln D^2 + \ln\frac{1}{R_1 R_2}\right) = \frac{Q}{\pi\varepsilon_0}\left(\frac{1}{2}\ln\frac{D^2}{R_1 R_2}\right) =$$

$$= \frac{Q}{\pi\varepsilon_0} \ln\frac{D}{\sqrt{R_1 R_2}}$$

Thus $C = \frac{Q}{\Delta v} = \frac{\pi\varepsilon_0}{\ln\frac{D}{\sqrt{R_1 R_2}}}$

Since $R_1 = R_2 = R$

$$C = \frac{\pi\varepsilon_0}{\ln\frac{D}{R}}$$

\- Esn. 4 -

a) ~~Clear~~ The line is not lossless (only $G=0$) and is long (≈ 300 km)

$$Z_w = \sqrt{\frac{Z_s}{Y_p}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{0.03+j0.4}{j5\cdot 10^{-6}}} = 283.2 \angle -2.14^\circ$$

$$= 2.83\cdot 10^2 - j1.06\cdot 10 =$$

the line is slightly capacitive as expected.

$$= 283 - j10.6$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = 0.0001 + j0.0014 =$$

$$= 0.00145 \angle 87.85^\circ \ \ Km^{-1}$$

b)

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \overset{T}{\begin{bmatrix} Ch\gamma l & Z_w Sh\gamma l \\ \frac{1}{Z_w} Sh\gamma l & Ch\gamma l \end{bmatrix}} \cdot \begin{bmatrix} V(l) \\ I(l) \end{bmatrix}$$

$$s = \gamma l = 0.0159 + j0.426 = 0.4249 \angle 87.85^\circ$$

$$Ch\gamma l = 0.9113 + j0.0065 = 0.9114 \angle 0.4117^\circ$$

$$Sh\gamma l = 0.0145 + j0.4120 = 0.4122 \angle 87.98^\circ$$

$$Z_w Sh\gamma l = 8.4672 + j1.1645\cdot 10^2 = 116.76 \angle 85.84^\circ$$

$$\frac{1}{Z_w} Sh\gamma l = -0 + j0.0015 = j0.0015 \angle 90.13$$

-10^{-4}. This explains the angle slightly higher than 90°

~~Thus~~

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = \begin{bmatrix} 345 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = T \begin{bmatrix} 345 \\ 0 \end{bmatrix}$$

Thus

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} 314.41 + j2.26 \\ j0.5 \end{bmatrix} \Rightarrow |V(0)| = 314.42 \text{ kV}$$

c)

$$Z_s = Z_w Sh\gamma l = 8.47 + j111.45$$

$$Y_{1p} = Y_{2p} = \left(\frac{1}{Z_w} Th\left(\frac{\gamma l}{2}\right)\right) \simeq j7.616\cdot 10^{-4}$$