

**FINAL Exam****Autumn 2016****Duration: 90 minutes**

- It is advisable to take knowledge of the entire text before answering any question.
- Applicants must respect the used notation and specify in each case the question number.
- Most attention will be given to the clarity of writing, presentation, the diagram and the presence of measurement unit

Results will be put in frames

**Exercises are independent**

**Documentation: An A4 double face is authorized, Calculator authorized, phone forbidden**

**Exercise 1:**

A particle of unit mass moves along the x-axis subject to a force  $u$ .

This system is described by a partial differential equation  $\ddot{y} = u$

- 1- Rewrite the system equation under the state space model by defining  $x_1$  and  $x_2$ .

This particle should be moved from rest position at initial time from origin ( $x_1(0)=x_2(0)=0$ ) to the desired position after 1 unit of time such that ( $x_1(1)=1, x_2(1)=0$ ).

It is required to find the optimal control  $u^*$  to move this system from the original position to the final position with a minimum of energy.

- 2- Give the optimization criterion
- 3- Write the Hamiltonian equation
- 4- Give the canonical equations of Hamilton
- 5- Solve the previous system in order to find the expression of the optimal control
- 6- Use the given problem data to find out the different constants (all constants should be found)

**Exercise 2:**

A user has decided to install a photovoltaic system. This will be positioned on a south facing roof of his house in Belfort. The roof surface available is shown below in Figure 1.

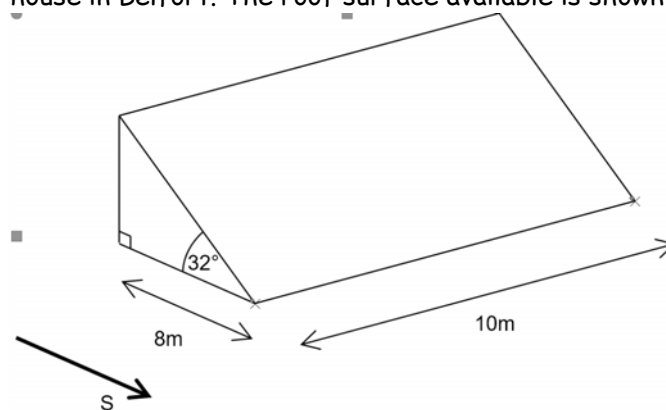


Figure 1 Roof for Photovoltaic System

The BP Solar 3160 module has been selected for the system (see Table 1). A number of Inverters have also been identified (Table 2).

1. How many modules are required to make best use of the roof surface area available?
2. Which inverter would be most appropriate? To calculate this you must consider the number of modules in a string and the number of strings (i.e. the number of modules in series and the number of parallel lines of series-connected modules).
3. What is the cost of the equipment for the system (modules + inverter)?
4. This user agreed to install the system by himself and to integrate it to the building roof, so there are no installation costs. In order to recover the investment in 10 years, what price should be charged per kWh of electricity?
5. Considering the updated price of the solar electricity in France (end 2016 around 24c€/kWh). Compare with the price that EDF pay for this energy and re-calculate the Return On Investment (ROI) period.

Annual Solar Irradiation for Belfort on a south-facing surface inclined at 32°: 1300kWh/m<sup>2</sup>.year

Table 1 Data for PV Module


Photovoltaic Module	Module photo	$P_{MPP}$ (W) $\equiv$ (Wp)	$V_{MPP}$ (V)	$V_{OC}$ (V)	$I_{MPP}$ (A)	$I_{SC}$ (A)	Width (m)	Height (m)	Cost (€/Wp installed)
<b>Rooftop Solution:</b> BP Solar BP 3160		160	35.1	44.2	4.55	4.8	0.79	1.59	3

Table 2 Inverter Datasheet

Inverter	Max DC Power (W)	Max DC Voltage (V)	Cost (€)
A) SB 8_640	8000	640	8000
B) SB 8_760	8000	760	12000
C) SB 11_980	11000	980	16000
D) SB 11_1100	11000	1100	22000