ER57 18 gennaio 2024

Let's define

$$\delta = \delta_1 - \delta_2$$

~~Thus if the current I_{12} is lagging δ_1~~

we have then

$$I_{12} = \frac{V_1 - V_2}{jX}$$

Then $$P_2 + jQ_2 = V_2 I_2^* = V_2 \left(\frac{V_1 - V_2}{jX} \right)^* = \frac{V_2 V_1^* - V_2 V_2^*}{-jX} =$$

$$= \frac{V_2 V_1^* - V_2 V_2^*}{-jX} = \frac{|V_2| e^{+j\delta_2} |V_1| e^{-j\delta_1} - |V_2| e^{j\delta_2} |V_2| e^{-j\delta_2}}{-jX} =$$

$$= \frac{|V_1||V_2| e^{-j\delta} - |V_2|^2}{-jX} = \left(-\frac{1}{j} = j \right) =$$

$$= \frac{j|V_1||V_2| e^{-j\delta} - j|V_2|^2}{X} = \frac{j|V_1||V_2|[\cos\delta - j\sin\delta] - j|V_2|^2}{X} =$$

$$= \frac{|V_1||V_2| \sin\delta + j(|V_1||V_2| \cos\delta - |V_2|^2}{X} =$$

$$= \frac{|V_1||V_2| \sin\delta}{X} + j\left(\frac{|V_1||V_2| \cos\delta - |V_2|^2}{X} \right)$$

3) Thus $$P_2 = \frac{|V_1||V_2| \sin\delta}{X}$$

4) $$Q_2 = \frac{|V_1||V_2| \cos\delta - |V_2|^2}{X}$$

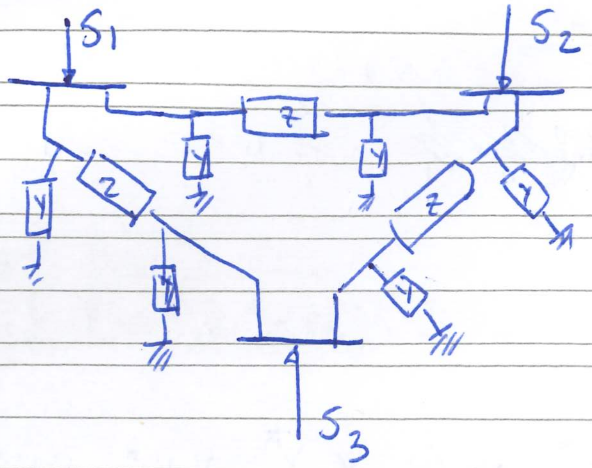
1) Since there is no real loss in the line $P_1 = P_2 = \frac{|V_1||V_2| \sin\delta}{X}$

2) Q_1 is easily found considering $S_1 = P_1 + jQ_1 = V_1 I_1^* = V_1 \left(\frac{V_1 - V_2}{jX} \right)^* =$

$\delta = \delta_1 - \delta_2$ $$= \frac{V_1 V_1^* - V_1 V_2^*}{-jX} = \frac{|V_1|^2 - |V_1||V_2| e^{j\delta}}{-jX} = \frac{-j|V_1|^2 - j|V_1||V_2|(\cos\delta + j\sin\delta)}{X} =$$

$$= \underbrace{\frac{|V_1||V_2| \sin\delta}{X}}_{= P_1} + j\,\frac{|V_1|^2 - |V_1||V_2| \cos\delta}{X} \Rightarrow Q_1 = \frac{|V_1|^2 - |V_1||V_2| \cos\delta}{X}$$

Question 2



$$Y_{BUS} = \begin{bmatrix} 2Y + \frac{2}{Z} & -\frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & 2Y + \frac{2}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & -\frac{1}{Z} & 2Y + \frac{2}{Z} \end{bmatrix} := \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

we know that

$$I_1 = \frac{S_1^*}{V_1^*} = y_{11} V_1 + y_{12} V_2 + y_{13} V_3$$

$$I_2 = \frac{S_2^*}{V_2^*} = y_{21} V_1 + y_{22} V_2 + y_{23} V_3$$

$$I_3 = \frac{S_3^*}{V_3^*} = y_{31} V_1 + y_{32} V_2 + y_{33} V_3$$

i) But

$$S_1^* = P_1 - jQ_1 = y_{11} V_1 V_1^* + y_{12} V_1^* V_2 + y_{13} V_1^* V_3 = V_1^* \sum_{k=1}^{3} y_{1k} V_k$$

$$S_2^* = P_2 - jQ_2 = y_{21} V_1 V_2^* + y_{22} V_2 V_2^* + y_{23} V_3 V_2^* = V_2^* \sum_{k=1}^{3} y_{2k} V_k$$

$$S_3^* = P_3 - jQ_3 = y_{31} V_1 V_3^* + y_{32} V_2 V_3^* + y_{33} V_3 V_3^* = V_3^* \sum_{k=1}^{3} y_{3k} V_k$$

These are the LFE in complex form

2) since $V_i = |V_i| e^{j\delta_i}$ $\quad (i=1,2,3)$

$y_{ij} = |y_{ij}| \angle \gamma_{ij}$ we have

$$P_i - jQ_i = \sum_{k=1}^{3} |y_{ik}||V_i||V_k| e^{j(\delta_k - \delta_i + \gamma_{ik})} \qquad i=1,2,3.$$

Thus

$$P_1 = P_{G1} - P_{D1} = |y_{11}||V_1||V_1| \cos(\gamma_{11}) + |y_{12}||V_1||V_2| \cos(\delta_2 - \delta_1 + \gamma_{12}) +$$
$$+ |y_{13}||V_1||V_3| \cos(\delta_3 - \delta_1 + \gamma_{13})$$

$$P_2 = P_{G2} - P_{D2} = |y_{21}||V_2||V_1| \cos(\delta_1 - \delta_2 + \gamma_{21}) + |y_{22}||V_2|^2 \cos(\gamma_{22}) +$$
$$+ |y_{23}||V_2||V_3| \cos(\delta_3 - \delta_2 + \gamma_{32})$$

$$P_3 = P_{G3} - P_{D3} = |y_{31}||V_3||V_1| \cos(\delta_1 - \delta_3 + \gamma_{31}) + |y_{32}||V_3||V_2| \cos(\delta_2 - \delta_3 + \gamma_{32})$$
$$+ |y_{33}||V_3|^2 \cos(\gamma_{33})$$

And

$$-Q_1 = Q_{G1} - Q_{D1} = |y_{11}||V_1|^2 \sin(\gamma_{11}) + |y_{12}||V_1||V_2| \sin(\delta_2 - \delta_1 + \gamma_{12}) + |y_{13}||V_1||V_3| \cos(\delta_3 - \delta_1 + \gamma_{13})$$

$$-Q_2 = Q_{G2} - Q_{D2} = |y_{21}||V_1||V_2| \sin(\delta_1 - \delta_2 + \gamma_{21}) + |y_{22}||V_2|^2 \sin(\gamma_{22}) + |y_{23}||V_2||V_3| \sin(\delta_3 - \delta_2 - \gamma_{32})$$

$$-Q_3 = Q_{G3} - Q_{D3} = |y_{31}||V_3||V_1| \sin(\delta_1 - \delta_3 + \gamma_{31}) + |y_{32}||V_3||V_2| \sin(\delta_2 - \delta_3 + \gamma_{32}) + |y_{33}||V_3|^2 \sin(\gamma_{33})$$

Question 3

1) load on G1 = x MW

load on G2 = 300 − x MW

Reduction in frequency Δf

$$\Delta f = -R_1 \Delta P_{g1} = -\frac{0.04 \cdot 50}{100} x$$

$$\Delta f = -R_2 \Delta P_{g2} = -\frac{0.05 \cdot 50}{300} (300 - x)$$

Since the frequency is the same at steady-state, we have

$$-R_1 \Delta P_{g1} = -R_2 \Delta P_{g2} \Longrightarrow$$

$$\frac{0.04 \cdot 50}{100} x = \frac{0.05 \cdot 50}{300} (300 - x)$$

$$0.02 x = 0.0083 (300 - x)$$

$$x = 88.34 \qquad 300 - x = 211.6$$

Thus generator 1 will supply 88.34 MW
" 2 " " 211.6 MW

2) The system frequency is therefore

$$f = 50 - 0.02 \cdot 88.34 = 48.23 \text{ Hz}$$

Q 4

The ~~two~~ generators should pick up 20 MW and 200 MW.

$\left(\frac{60}{600} = \frac{1}{10} \right.$ so $\left. \frac{20}{200} = \frac{1}{10} \right)$ or

x = load of generator 1

y = load of generator 2

Since we want the two generators to share the load in the same ratio of the nominal power $(60/600 = \frac{1}{10})$, then we must have:

$$\begin{cases} \frac{x}{y} = \frac{1}{10} \\ x + y = 220 \end{cases} \rightarrow x = 20 \quad y = 200$$

From the formula $\Delta P_T = -\frac{1}{R} \Delta f$ we have

$$R_1 = - \frac{\Delta f}{\Delta P_{T1}} = - \frac{-0.5}{20} = 0.025 \text{ Hz/MW}$$

$$R_2 = - \frac{\Delta f}{\Delta P_{T2}} = - \frac{-0.5}{200} = 0.0025 \text{ Hz/MW}$$

If we express R in pu H / pu MW we have

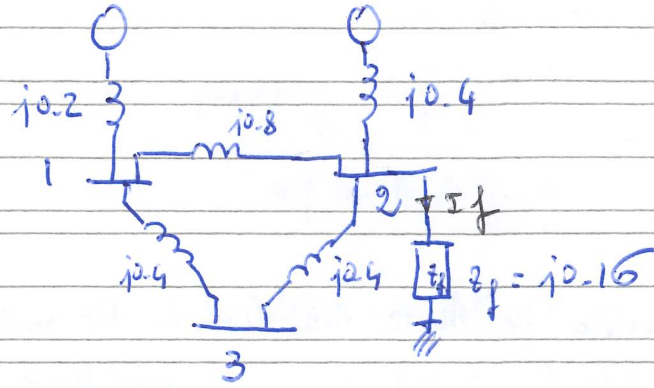
1)
$$R_1 = 0.025 \frac{60}{50} = 0.03 \text{ pu Hz/pu MW}$$

$$R_2 = 0.0025 \frac{600}{50} = 0.03 \text{ pu Hz/MW}$$

2) Generators working in parallel on the same network ought to have the same regulation expressed in pu Hz/pu MW of their own rating, in order to share load changes in proportion to their own ratings.

— Ex. 5 —

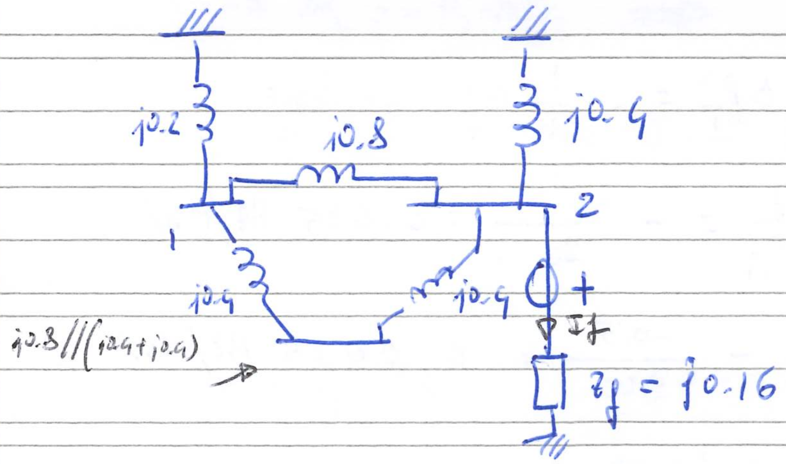
The ~~sit~~ situation is depicted in fig. below



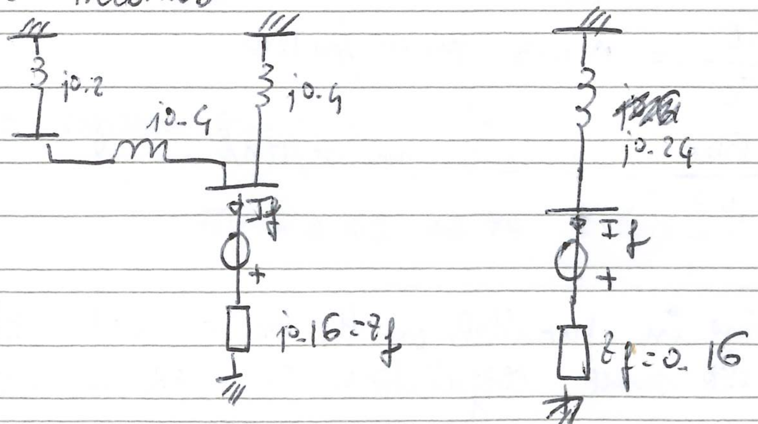
$V_2(0) = 1$ pu

$V_1(0) = V_2(0) = V_3(0) = 1$ pu

The Thévenin's equivalent circuit is



This becomes



$= (j0.4) // (j0.2 + j0.4) =$

$= j0.24$

1) $I_f = \dfrac{V_2(0)}{j0.24 + j0.16} = \dfrac{1}{j0.24 + j0.16} = -j\,2.5$ pu

Thus $I_{G1} = \dfrac{j0.4}{j0.4 + j0.6} I_f = \dfrac{j0.4}{j0.4+j0.6}(-j2.5) = -j$ pu

$$I_{G2} = \frac{j0.6}{j0.4+j0.6}(-j2.5) = -j1.5$$

2) The bus voltages changes are

$$\Delta V_1 = -(j0.2)(-j) = -0.2 \text{ pu}$$

$$\Delta V_2 = -(j0.4)(-j1.5) = -0.6 \text{ pu}$$

$$\Delta V_3 = \Delta V_1 - \frac{I_{G1}}{2} j0.4 = -0.2 - (j0.4)\left(-\frac{j}{2}\right) = -0.4 \text{ pu}$$

Thus the bus voltages are

$$V_1 = V_1 + \Delta V_1 = 1 - 0.2 = 0.8 \text{ pu}$$

$$V_2 = V_2 + \Delta V_2 = 1 - 0.6 = 0.4 \text{ pu}$$

$$V_3 = V_3 + \Delta V_3 = 1 - 0.4 = 0.6 \text{ pu}$$

3) The short-circuit currents in the lines are :

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{0.8 - 0.4}{j0.8} = -j0.5 \text{ pu}$$

$$I_{13} = \frac{V_1 - V_3}{Z_{13}} = \frac{0.8 - 0.6}{j0.4} = -j0.5 \text{ pu}$$

$$I_{32} = I_{13} = \frac{0.6 - 0.4}{j0.4} = -j0.5 \text{ pu}$$

obviously all three line currents are the same