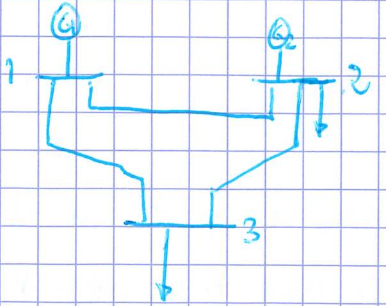
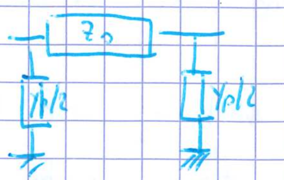
ER66 Médian 2024 5 novembre

SOLUTION

_Ex 1_



a) Each line is modelled as follows (π-model)



$$Y_s = \frac{1}{Z_s}$$

Then

$$y_{11} = Y_p/2 + \frac{1}{Z_s} + Y_p/2 + \frac{1}{Z_s} = y_{22} = y_{33} = Y_p + \frac{2}{Z_s}$$

off diagonal elements $\underset{(i \neq j)}{y_{ij}} = -\frac{1}{Z_s}$

Then Y_{bus} is

$$Y_{bus} = \begin{bmatrix} Y_p + \frac{2}{Z_s} & -\frac{1}{Z_s} & -\frac{1}{Z_s} \\ -\frac{1}{Z_s} & Y_p + \frac{2}{Z_s} & -\frac{1}{Z_s} \\ -\frac{1}{Z_s} & -\frac{1}{Z_s} & Y_p + \frac{2}{Z_s} \end{bmatrix}$$

b) For bus 1 $S_1 = V_1 I_1^* = P_{G1} + jQ_{G1} = V_1 I_1^*$ (there is no load)

$P_1 = P_G \quad Q_1 = Q_G$

For bus 2 $S_2 = \left(P_{G2} + jQ_{G2}\right) - \left(P_{L2} + jQ_{L2}\right) = \left(P_{G2} - P_{L2}\right) + j\left(Q_{G2} - Q_{L2}\right)$ (There is a generator and a load)

$$P_2 = P_{G2} - P_{L2} \qquad Q_2 = Q_{G2} - Q_{L2}$$

For bus 3 $S_3 = V_3 I_3^* = (0 - P_{L3}) + j(0 - Q_{L3}) =$

$= -P_{L3} - jQ_{L3}$

Thus
$P_3 = -P_{L3}$
$Q_3 = -Q_{L3}$

c)

- Since bus 2 is connected to a generator, it can be a PV bus (P_2 is given, $|V_2|$ is given, we have to compute Q_2 (Q_{G2}) and δ_2
- Since bus 3 is connected to a load, it can be a PQ bus. P_3 and Q_3 are given, we have to compute $|V_3|$ and δ_3
- Bus 1 is a slack bus, ~~so~~ $|V_1|$ and δ_1 are known ($|V_1| = 1$, for example, and $\delta_1 = 0$), and we have to compute P_{G1} and Q_{G1}.

d) For each bus i, ~~we have~~ we have

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = Y_{bus} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I_1 = \frac{S_1^*}{V_1^*} = y_{11} V_1 + y_{12} V_2 + y_{13} V_3$$

$$I_2 = \frac{S_2^*}{V_2^*} = y_{21} V_1 + y_{22} V_2 + y_{23} V_3$$

$$I_3 = \frac{S_3^*}{V_3^*} = y_{31} V_1 + y_{32} V_2 + y_{33} V_3$$

Then, in complex form, we have

$$S_1^* = y_{11} V_1 V_1^* + y_{12} V_1^* V_2 + y_{13} V_1^* V_3$$

$$S_2^* = y_{21} V_2^* V_1 + y_{22} V_2^* V_2 + y_{23} V_2^* V_3$$

$$S_3^* = y_{31} V_3^* V_1 + y_{32} V_2^* V_2 + y_{33} V_3^* V_3$$

Since $V_i = |V_i| e^{j\delta_i}$ and $y_{ij} = |y_{ij}| e^{j\gamma_{ij}}$, we have

$$P_1 - jQ_1 = |y_{11}| e^{j\gamma_{11}} |V_1|^2 + |y_{12}| e^{j\gamma_{12}} |V_1| e^{-j\delta_1} |V_2| e^{j\delta_2} + |y_{13}| e^{j\gamma_{13}} |V_1| e^{-j\delta_1} |V_3| e^{j\delta_3}$$

$$= |y_{11}| e^{j\gamma_{11}} |V_1|^2 + |y_{12}||V_1||V_2| e^{j(\delta_2 - \delta_1 + \gamma_{12})} + |y_{13}||V_1||V_3| e^{j(\delta_3 - \delta_1 + \gamma_{13})}$$

Or

$$P_1 - jQ_1 = |y_{11}||V_1||V_1| e^{j(\delta_1 - \delta_1 + \gamma_{11})} + |y_{12}||V_1||V_2| e^{j(\delta_2 - \delta_1 + \gamma_{12})} + |y_{13}||V_1||V_3| e^{j(\delta_3 - \delta_1 + \gamma_{13})}$$

Remembering that $e^{j\vartheta} = \cos\vartheta + j\sin\vartheta$, we can separate the real and the imaginary parts.

$$P_1 = |y_{11}||V_1||V_1| \cos(\delta_1 - \delta_1 + \gamma_{11}) + |y_{12}||V_1||V_2| \cos(\delta_2 - \delta_1 + \gamma_{12}) + |y_{13}||V_1||V_3| \cos(\delta_3 - \delta_1 + \gamma_{13})$$

$$-Q_1 = |y_{11}||V_1||V_1| \sin(\delta_1 - \delta_1 + \gamma_{11}) + |y_{12}||V_1||V_2| \sin(\delta_2 - \delta_1 + \gamma_{12}) + |y_{13}||V_1||V_3| \sin(\delta_3 - \delta_1 + \gamma_{13})$$

In compact form:

$$P_1 = \sum_{k=1}^{3} |y_{1k}||V_1||V_k| \cos(\delta_k - \delta_1 + \gamma_{1k})$$

$$-Q_1 = \sum_{k=1}^{3} |y_{1k}||V_1||V_k| \sin(\delta_k - \delta_1 + \gamma_{1k})$$

The same is for bus 2 and bus 3

$$P_2 = \sum_{k=1}^{3} |y_{2k}||V_2||V_k| \cos(\delta_k - \delta_2 + \gamma_{2k})$$

$$-Q_2 = \sum_{k=1}^{3} |y_{2k}||V_2||V_k| \sin(\delta_k - \delta_2 + \gamma_{2k})$$

$$P_3 = \sum_{k=1}^{3} |y_{3k}||V_3||V_k| \cos(\delta_k - \delta_3 + \gamma_{3k})$$

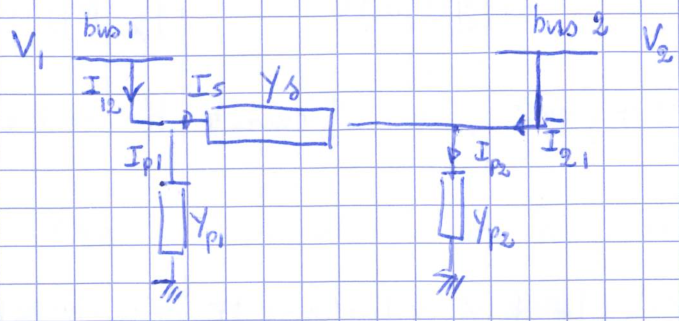
$$Q_3 = \sum_{k=1}^{3} |y_{3k}||V_3||V_k| \sin(\delta_k - \delta_3 + \gamma_{3k})$$

Or in general

$$\begin{cases} P_i = \sum_{k=1}^{3} |y_{ik}||V_i||V_k| \cos(\delta_i - \delta_k + \gamma_{ik}) \\ -Q_i = \sum_{k=1}^{3} |y_{ik}||V_i||V_k| \sin(\delta_i - \delta_k + \gamma_{ik}) \end{cases}$$

$i = 1, 2, 3$

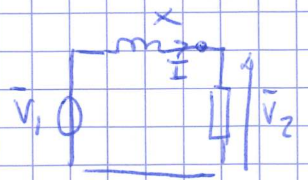
e) Once all voltages are known for all buses, the power flows on the various lines of the network are computed by using the π-model of the line.



Then $I_{12} = I_s + I_{p1} = (V_1 - V_2)Y_s + V_1 Y_{p1}$ $\quad I_{21} = -I_s + I_{p2} = (V_2 - V_1)Y_s + V_2 Y_{p2}$

Thus
$$S_{12} = P_{12} + jQ_{12} = V_1 I_{12}^* = V_1(V_1^* - V_2^*)Y_s^* + |V_1|^2 Y_{p1}^*$$
$$S_{21} = P_{21} + jQ_{21} = V_2 I_{21}^* = V_2(V_2^* - V_1^*)Y_s^* + |V_2|^2 Y_{p2}$$

- Ex 2 -



$S_2 = V_2 I^*$

but $I = \dfrac{V_1 - V_2}{jx}$ so $S_2 = V_2 \dfrac{(V_1 - V_2)^*}{-jx} =$

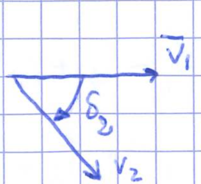
then

$$S_2 = j\frac{V_2 V_1^* - |V_2|^2}{x} \qquad \left(\frac{1}{j} = -j\right)$$

We impose $V_1 = |V_1| e^{j0}$ and $V_2 = |V| e^{-j\delta_2}$

That is we consider v_1 to be the reference voltage

so



Under these assumptions we have

$$S_2 = j\,\frac{|V_2|\,e^{-j\delta_2}\,|V_1| - |V_2|^2}{X}$$

Since $e^{-j\delta_2} = \cos\delta_2 - j\sin\delta_2$, we have:

$$S_2 = \frac{j\left(|V_1||V_2|\cos\delta_2 - j|V_1||V_2|\sin\delta_2\right) - j|V_2|^2}{X} =$$

$$= \frac{|V_1||V_2|\sin\delta_2}{X} + j\,\frac{|V_1||V_2|\cos\delta_2 - |V_2|^2}{X}$$

Thus
$$\begin{cases} P = \dfrac{|V_1|\,|V_2|\sin\delta}{X} \\[2ex] Q = \dfrac{|V_1|\,|V_2|\cos\delta - |V_2|^2}{X} \end{cases} \qquad \delta = \delta_2$$

$\delta = \delta_2$ is the angle between phasor $\bar{V}_1$ and $\bar{V}_2$; it is considered positive when $\bar{V}_1$ leads $\bar{V}_2$, so that $P > 0$.

Question 3

$$C_1 = 10 P_1 + 0.008 P_1^2 \quad \$/h$$

$$C_2 = 8 P_2 + 0.009 P_2^2 \quad \$/h$$

$P_T = 1500$ MW

1. Since there are no transmission losses, the optimal economical dispatch is obtained when $IC_1 = IC_2$

$$IC_1 = \frac{dC_1}{dP_1} = 10 + 0.016 P_1 \quad \$/MWh$$

$$IC_2 = \frac{dC_2}{dP_2} = 8 + 0.018 P_2 \quad \$/MWh$$

Remarking that $P_2 = P_T - P_1$, and that $IC_1 = IC_2$, we have

$$10 + 0.016 P_1 = 8 + 0.018 (P_T - P_1)$$

$$0.016 P_1 = 0.018 (1500 - P_1) - 2$$

$$0.016 P_1 = 27 - 0.018 P_1 - 2 = 25 - 0.018 P_1$$

$$0.034 P_1 = 25 \qquad P_1 = 735.28 \text{ MW}$$

$$P_2 = 1500 - P_1 = 764.72$$

2. $IC_1 = IC_2 = 10 + 0.016 P_1 = 21.76 \quad \$/MWh$

3. $C_T = C_1 + C_2 = 10 \cdot 735.28 + 0.008 (735.28)^2 + 8 \cdot 764.72 + 0.009 (764.72)^2 =$

$= 23{,}058 \quad \$/h$

– Question 4 –

a) $$G(s) = \frac{k_a k_e k_f}{(1+T_a s)(1+T_e s)(1+T_f s)}$$

Type 0, ~~ss~~ since there are no poles in the origin

b) The closed loop of the system, defining $K = k_a k_e k_f$, is

$$W(s) = \frac{K}{(1+T_a s)(1+T_e s)(1+T_f s) + K}$$

Then
$$\Delta y = W(s)\,\Delta u(s)$$

If $u(s)$ is a step input, then $u(s) = \frac{1}{s}$. The output would be therefore:

$$\Delta y(s) = W(s)\,\frac{1}{s}$$

In steady state $\Delta y(\infty) = \lim_{s\to 0} s\,\Delta y(s)$ (Final value theorem), so, calling Δy_{ss} the value at steady state $\Delta y(\infty)$

$$\Delta y_{ss} = \lim_{s\to 0}\left[s\,W(s)\,\frac{1}{s}\right] = W(0)$$

$$W(0) = \frac{K}{1+K}$$

The error between the input value 1 and the steady-state response is

$$\Delta e_{ss} = 1 - \frac{K}{1+K} = \frac{1}{1+K}$$

~~Since $\Delta e_{ss} \le 0.01$, $\frac{1}{1+K}$, we have $K \geq 99$~~

But Δe_{ss} is requested to be less than 1%, or 0.01, so:

$$\frac{1}{1+K} \le 0.01 \quad \text{or} \quad K \geq 99$$

$$G(s) = \frac{K_a}{(1+0.1s)(1+0.4s)(1+s)} = \qquad p_1 = -1 \quad p_2 = -2.5 \quad p_3 = -10$$

$$= \frac{K_a}{(0.1)(0.4)(s+10)(s+2.5)(s+1)} = \frac{25\,K_a}{(s+1)(s+10)(s+2.5)}$$

$q = n - m = 3$

$\gamma = 60, 0, -60$

$$\sigma_a = \frac{\Sigma p - \Sigma z}{3} = \frac{-13.5}{3} = -4.5$$

$$P(s) = (s+1)(s+10)(s+2.5) + 25K_a = s^3 + 13.5s^2 + 37.5s + 25(1+K_a)$$

s^3	1	37.5
s^2	13.5	$25(1+K_a)$
s^1	$37.5 - 0.66(1+K_a)$	
s^0	$25(1+K_a)$	

$$s^1 = \frac{13.5 \cdot 37.5 - 25(1+K_a)}{37.5} = 13.5 - 0.66(1+K_a)$$

We must have

$1 + K_a > 0 \qquad K_a > -1 \rightarrow K_a > 0$

$13.5 - 0.66 - 0.66K_a > 0 \qquad 12.83 - 0.66K_a > 0 \qquad K_a < 19.44$

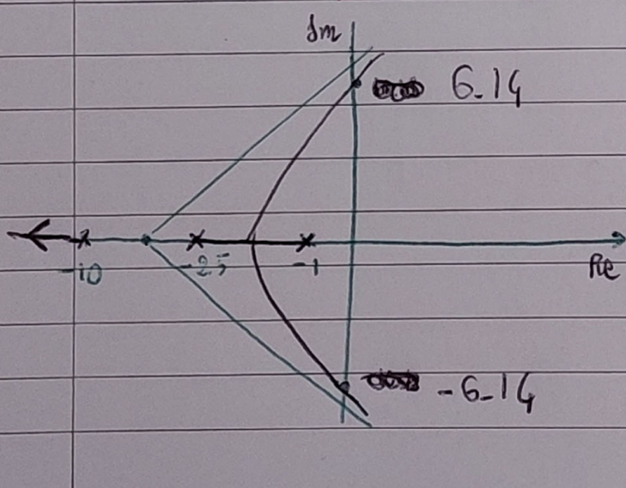
Thus $0 \leq K_a \leq 19.44$

$25(1+K_a) \cong 511$

Auxiliary equation:

$13.5s^2 + 511 = 0 \qquad s^2 + 37.8 = 0$

$s = \pm j\sqrt{37.8} = 6.14$



$\sigma_a = -4.5.$