12/01/15

MQ40 - Automne 2014

FINAL - Corrigé

1.1

$$I_G = \iint_{Section} r^2 \, ds$$

Coordonnées polaires

$$I_G = \int_0^R \int_0^{2\pi} r^2 \, r \, d\theta \, dr$$

$$\boxed{I_G = \frac{\pi R^4}{2}}$$

1.2

$$I_G = \iint_{Section} (y^2 + z^2) \, ds$$

$$I_G = \iint_{Section} y^2 \, ds + \iint_{Section} z^2 \, ds$$

$$I_G = 2 \iint_{Section} y^2 \, ds \quad \text{(compte-tenu des symétries)}$$

$$I_G = 2 \, I_z$$

$$\boxed{I_z = \frac{\pi R^4}{4}}$$

1.3 $S = \pi R^2$ $\boxed{S = 314 \text{ mm}^2}$

$$I_4 = \frac{\pi R^4}{2} \qquad \boxed{I_4 = 15708 \text{ mm}^4}$$

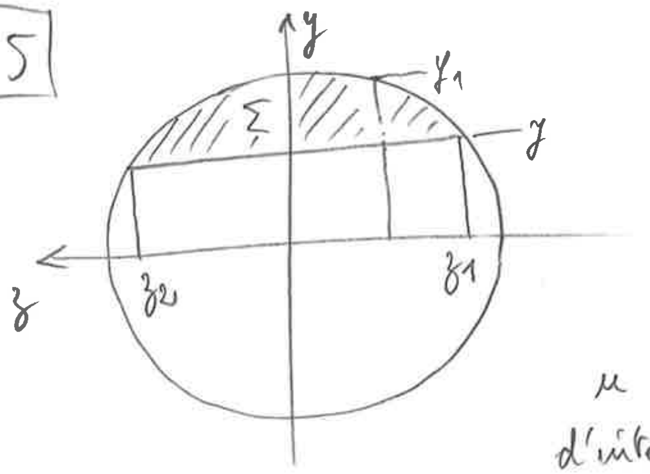
$$I_z = \frac{\pi R^4}{4} \qquad \boxed{I_z = 7854 \text{ mm}^4}$$

1.4 Théorème de Pythagore

$$\left(\frac{l(y)}{2}\right)^2 + y^2 = R^2$$

$$\boxed{l(y) = 2\sqrt{R^2 - y^2}}$$

1.5



$$\mu(y) = \iint_\Sigma y \, ds$$

$$\mu(y) = \int_{z_1}^{z_2} \int_y^{y_1} u \, du \, dz$$

u remplace y comme variable d'intégration, pour éviter toute confusion avec la borne d'intégration.

$$z_2 = -z_1 = \frac{l(y)}{2}$$

$$y_1 = \sqrt{R^2 - z^2}$$

$$\mu(y) = \int_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} \int_y^{\sqrt{R^2-z^2}} u \, du \, dz$$

$$\mu(y) = \int_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} \left[\frac{u^2}{2}\right]_y^{\sqrt{R^2-z^2}} dz$$

$$\mu(y) = \frac{1}{2}\int_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} (R^2 - z^2 - y^2)\, dz$$

$$\mu(y) = \frac{1}{2}\left\{(R^2-y^2)\left[z\right]_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} - \frac{1}{3}\left[z^3\right]_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}}\right\}$$

$$\mu(y) = \frac{1}{2}\left\{(R^2-y^2)\times 2\sqrt{R^2-y^2} - \frac{1}{3}\times 2 \times (R^2-y^2)^{\frac{3}{2}}\right\}$$

$$\mu(y) = (R^2-y^2)^{\frac{3}{2}}\left(1-\frac{1}{3}\right)$$

$$\boxed{\mu(y) = \frac{2}{3}(R^2-y^2)^{\frac{3}{2}}}$$

$\boxed{2}$ $\left\{\mathcal{C}_{cohésion}\right\} = \left\{\begin{matrix}\vec{F_G}\\ \vec{M_G}\end{matrix}\right\}$

$\vec{F_G} = \vec{F_E}$ $\qquad \vec{F_G} = \begin{pmatrix}F_{Ex}\\ F_{Ey}\\ 0\end{pmatrix} \longrightarrow \left\{\begin{matrix}\boxed{2.1} & \boxed{N = F_{Ex}}\\ \boxed{2.2} & \boxed{T_y = F_{Ey}}\end{matrix}\right.$

$$\vec{M_G} = \vec{M_E} + \vec{GE} \wedge \vec{F_E}$$

$$\vec{M_G} = \begin{pmatrix}M_{Ex}\\ 0\\ 0\end{pmatrix} + \left[\begin{pmatrix}D\\ 0\\ 0\end{pmatrix} \wedge \begin{pmatrix}F_{Ex}\\ F_{Ey}\\ 0\end{pmatrix}\right]$$

$\vec{M_G} = \begin{pmatrix}M_{Ex}\\ 0\\ DF_{Ey}\end{pmatrix} \longrightarrow \left\{\begin{matrix}\boxed{2.3} & \boxed{M_x = M_{Ex}}\\ \boxed{2.4} & \boxed{M_z = D\,F_{Ey}}\end{matrix}\right.$

2,5

$$\boxed{\begin{aligned} N &= 1000\,N \\ T_y &= 300\,N \\ M_x &= 50\,Nm \\ M_z &= 60\,Nm \end{aligned}}$$

3.1

$$\boxed{\sigma_{traction} = \frac{N}{S}}$$

3.2

$$\sigma_{traction} = \frac{1000}{314}$$

$$\boxed{\sigma_{traction} = 3{,}18\ MPa}$$

4.1

$$\boxed{\sigma_{flexion} = -\frac{M_z}{I_z}\,y}$$

4.2

G_1 : $y = R$ $\qquad \sigma_{flexion} = -\dfrac{60.10^3}{7854} \times 10$

G_2 et G_4 : $y = 0$ $\qquad \sigma_{flexion} = 0$

G_3 : $y = -R$ $\qquad \sigma_{flexion} = \dfrac{60,10^3}{7854} \times 10$

	G_1	G_2	G_3	G_4
$\sigma_{flexion}$, en MPa	-76,39	0	76,39	0

5.1

$$\boxed{\tau_{torsion} = \frac{M_x}{I_G} r}$$

5.2

$$\tau_{torsion} = \frac{50.10^3 \times 10}{15708} \quad (\text{pour } r = R)$$

$$\boxed{\tau_{torsion} = 31,83 \text{ MPa}}$$

6.1

$$\tau_{tranchant} = \frac{T_y \, \mu(y)}{I_z \, b(y)}$$

$$\tau_{tranchant} = \frac{T_y}{I_z} \times \frac{2}{3} \frac{(R^2-y^2)^{3/2}}{2(R^2-y^2)^{1/2}}$$

$$\boxed{\tau_{tranchant} = \frac{T_y (R^2-y^2)}{3 I_z}}$$

Autre expression :

$$I_z = \frac{\pi R^4}{4} = \frac{S R^2}{4}$$

$$\tau_{tranchant} = \frac{4 T_y}{3S}\left(1 - \frac{y^2}{R^2}\right)$$

6.2

G_1 : $y = R$ $\qquad \tau_{tranchant} = 0$

G_2 et G_4 : $y = 0$ $\qquad \tau_{tranchant} = \dfrac{300 \times 10^2}{3 \times 7854}$

G_3 : $y = -R$ $\qquad \tau_{tranchant} = 0$

	G_1	G_2	G_3	G_4
$\tau_{tranchant}$, en MPa	0	1,27	0	1,27

$\boxed{7}$ * Contrainte normale σ

$$\sigma = \sigma_{traction} + \sigma_{flexion}$$

$$(\sigma) = \begin{pmatrix} \sigma & \tau_3 & \tau_2 \\ \tau_3 & 0 & 0 \\ \tau_2 & 0 & 0 \end{pmatrix}$$

* Contrainte de cisaillement τ_2

Appliquée dans la direction z

$\tau_2 = +\tau_{torsion}$ en G_1

$\tau_2 = 0$ en G_2 et G_4

$\tau_2 = -\tau_{torsion}$ en G_3

* Contrainte de cisaillement τ_3

Appliquée dans la direction y

$\tau_3 = 0$ en G_1 et G_3

$\tau_3 = -\tau_{torsion} + \tau_{tranchant}$ en G_2

$\tau_3 = +\tau_{torsion} + \tau_{tranchant}$ en G_4

$\boxed{7.1}$ $G_1 : (\sigma) = \begin{pmatrix} -73,21 & 0 & 31,83 \\ 0 & 0 & 0 \\ 31,83 & 0 & 0 \end{pmatrix}$

$\boxed{7.2}$ $G_2 : (\sigma) = \begin{pmatrix} 3,18 & -30,56 & 0 \\ -30,56 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\boxed{7.3}$ $G_3 : (\sigma) = \begin{pmatrix} 79,58 & 0 & -31,83 \\ 0 & 0 & 0 \\ -31,83 & 0 & 0 \end{pmatrix}$

$\boxed{7.4}$ $G_4 : (\sigma) = \begin{pmatrix} 3,18 & 33,10 & 0 \\ 33,10 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\boxed{7.5}$ G_1 et G_3 : $\sigma_{VM} = \sqrt{\sigma^2 + 3\tau_2^2}$

G_2 et G_4 : $\sigma_{VM} = \sqrt{\sigma^2 + 3\tau_3^2}$

	G_1	G_2	G_3	G_4
σ_{VM}, en MPa	91,65	53,02	96,81	57,43

$\boxed{7.6}$ Limite : $\frac{2}{3} \times 100 = 67$ MPa dépassée en G_1 et G_3

Le dimensionnement n'est pas correct.