08/11/13

MEDIAN MQ40 – MQ43

Réponses

1.1 $\boxed{\omega = -\frac{V}{a}}$

1.2 $$\boxed{\{V\}_G = \begin{Bmatrix} 0 & V \\ 0 & 0 \\ -\frac{V}{a} & 0 \end{Bmatrix}}$$

1.3 $\vec{V}_A = \vec{V}_G + \vec{AG} \wedge \vec{\Omega}$ $$\boxed{\vec{V}_A = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

1.4 $\vec{V}_B = \vec{V}_G + \vec{BG} \wedge \vec{\Omega}$ $$\boxed{\vec{V}_B = \begin{pmatrix} -V \\ 0 \\ 0 \end{pmatrix}}$$

1.5 $\vec{V}_P = \vec{V}_G + \vec{PG} \wedge \vec{\Omega}$ avec $\vec{PG} = \begin{pmatrix} -2a\sin\theta \\ -2a\cos\theta \\ 0 \end{pmatrix}$

$$\boxed{\vec{V}_P = \begin{pmatrix} V + 2V\sin\theta \\ -2V\cos\theta \\ 0 \end{pmatrix}}$$

1.6 $\boxed{\theta = -\frac{V}{a}t}$

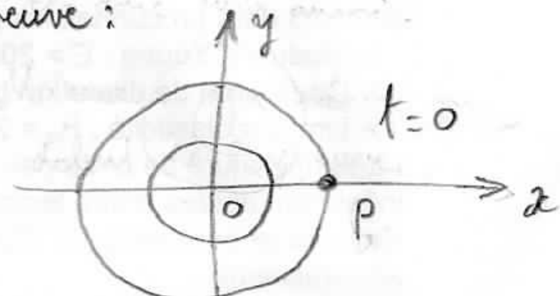
1.7 $\begin{cases} x'(t) = V - 2V \sin \dfrac{Vt}{a} \\ y'(t) = -2V \cos \dfrac{Vt}{a} \end{cases}$

Intégration avec conditions initiales $\begin{cases} x(0) = 2a \\ y(0) = 0 \end{cases}$

L'énoncé ne précise pas que G est en O à $t=0$

Vu au début de l'épreuve :

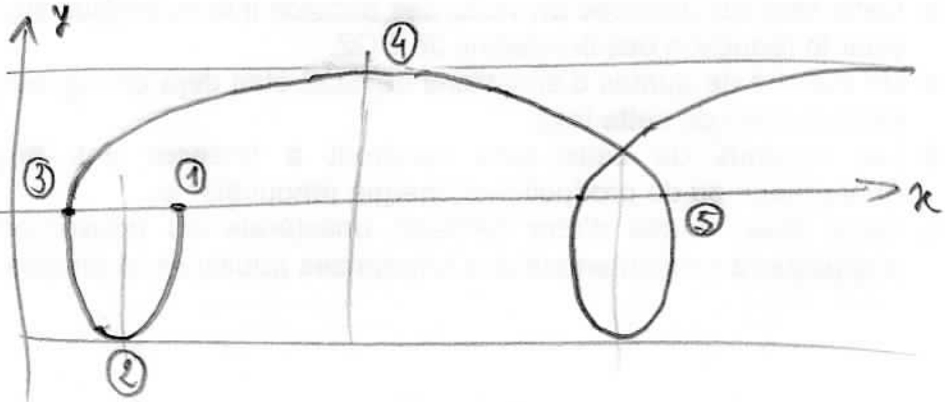
$$\boxed{\begin{cases} x(t) = Vt + 2a \cos \dfrac{Vt}{a} \\ y(t) = -2a \sin \dfrac{Vt}{a} \end{cases}}$$



1.8 ① $t=0$ $\begin{cases} x = 2a \\ y = 0 \end{cases}$ ② $t = \dfrac{\pi a}{2V}$ $\begin{cases} x = \dfrac{\pi a}{2} \\ y = -2a \end{cases}$

③ $t = \dfrac{\pi a}{V}$ $\begin{cases} x = a(\pi - 2) \\ y = 0 \end{cases}$ ④ $t = \dfrac{3\pi a}{2V}$ $\begin{cases} x = \dfrac{3\pi a}{2} \\ y = 2a \end{cases}$

⑤ $t = \dfrac{2\pi a}{V}$ $\begin{cases} x = 2a(\pi + 1) \\ y = 0 \end{cases}$



$\boxed{2.1}$ Cylindre plein : $I = \frac{MR^2}{2}$. $I = \rho\frac{\pi L R^4}{2}$

Petit disque : $I_1 = \rho\frac{\pi a^5}{2}$ $(R = L = a)$

2 grands disques : $I_2 = \frac{32}{2}\rho\pi a^5$ $(R = L = 2a)$

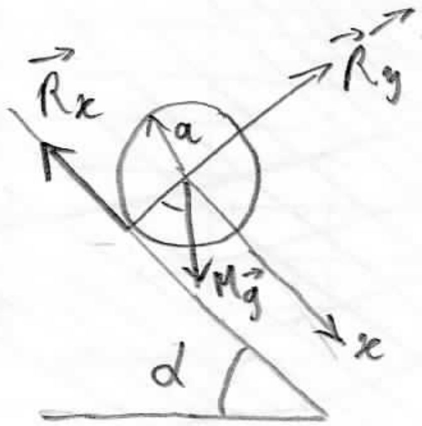
$I = \frac{33}{2}\rho\pi a^5$

Masses : Petit disque $m_1 = \rho\pi a^3$; 2 grands disques $m_2 = 8\rho\pi a^3$; $M = 9\rho\pi a^3$

$$\rho = \frac{M}{9\pi a^3}$$

$$\boxed{I = \frac{11}{6}Ma^2}$$

$\boxed{2.2}$



$$\boxed{Mg\sin\alpha - R_x = M x_G''}$$

$\boxed{2.3}$ $\boxed{-R_x a = I\theta''}$

$\boxed{2.4}$ $\boxed{x_G'' = -a\theta''}$

$\boxed{2.5}$ 2.2 $\longrightarrow$ $R_x = M(g\sin\alpha - x_G'')$

2.3 $\longrightarrow$ $-M(g\sin\alpha - x_G'')\,a = I\theta''$

2.1 + 2.4 $\longrightarrow$ $-M(g\sin\alpha - x_G'')\,a = \frac{11}{6}Ma^2\left(-\frac{x_G''}{a}\right)$

$$\boxed{x_G'' = \frac{6}{17}g\sin\alpha}$$

2.6 $x_G = \frac{3}{17} g \sin\alpha \, t^2$ $\qquad x'_G = \frac{6}{17} g \sin\alpha \, t$

$L = \frac{3}{17} g \sin\alpha \, t_1^2$

$$\boxed{t_1 = \sqrt{\frac{17 L}{3 g \sin\alpha}}}$$

2.7 $V_1 = x'_G(t_1)$

$V_1 = \frac{6}{17} g \sin\alpha \sqrt{\frac{17 L}{3 g \sin\alpha}}$

$$\boxed{V_1 = 2\sqrt{\frac{3 g \sin\alpha \, L}{17}}}$$

2.8 $E_c = \frac{1}{2} M x_G'^2 + \frac{1}{2} I \theta'^2$

En $x = L$ $\quad E_c = \frac{1}{2} M V_1^2 \left(1 + \frac{11}{6} a^2 \times \frac{1}{a^2}\right)$

$E_c = \frac{17}{12} M V_1^2$

$$\boxed{E_c = M g L \sin\alpha}$$

Autre solution : ΔE_c = travail du poids $= M g L \sin\alpha$
(théorème de l'énergie cinétique)

2.9 $\left\{\begin{array}{l} \text{Sur le rail } \theta' = -\frac{V}{a} \text{ et } E_c = \frac{17}{12} M V_1^2 \\ \text{Sur le plan horizontal : } \theta' = -\frac{V}{2a} \text{ et } E_c = \frac{1}{2} M V_2^2 \left(1 + \frac{11}{6} a^2 \times \frac{1}{4a^2}\right) \end{array}\right.$

Conservation de $E_c \rightarrow \frac{17}{12} M V_1^2 = \frac{35}{48} M V_2^2 \rightarrow \frac{V_2^2}{V_1^2} = \frac{4 \times 17}{35}$

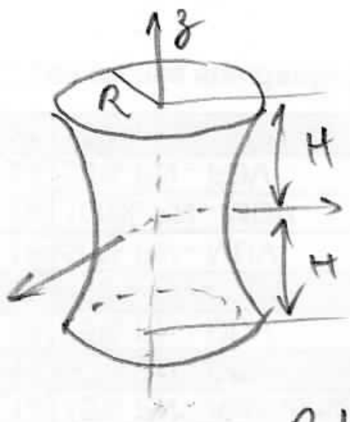
$\boxed{\frac{V_2}{V_1} = 139\%}$ Si l'énergie cinétique est conservée, la roue accélère.

3.1 $\boxed{r_{max}(z) = R\sqrt{\dfrac{c^2+z^2}{c^2+H^2}}}$

$r_{max}(-H) = r_{max}(H) = R$

$r_{max}(0) = \dfrac{Rc}{\sqrt{c^2+H^2}} < R$

3.2



3.3 $c = 0$ $\quad r_{max} = \dfrac{R|z|}{H}$ Solide = 2 cônes

3.4 $c \to +\infty$ $\quad r_{max} \to R$ Solide → cylindre

3.5 $$M = \rho \int_{-H}^{H}\int_0^{2\pi}\int_0^{r_{max}} r\,dr\,d\theta\,dz$$

$$\boxed{M = \frac{2\pi\rho R^2 H(3c^2+H^2)}{3(c^2+H^2)}} \quad \longrightarrow \rho = \frac{3M(c^2+H^2)}{2\pi R^2 H(3c^2+H^2)}$$

3.6 $c = 0$ $\quad \boxed{M = \frac{2}{3}\pi\rho R^2 H}$ Vu en cours ; masse d'un cône $= \frac{1}{3}\pi\rho R^2 H$

$c \to \infty$ $\quad \boxed{M \to 2\pi\rho R^2 H}$ masse d'un cylindre

3.7 $$I_z = \rho \int_{-H}^{H}\int_0^{2\pi}\int_0^{r_{max}} r^3\,dr\,d\theta\,dz$$

$$I_z = \frac{\pi\rho R^4 H(15c^4+3H^4+10c^2H^2)}{15(c^2+H^2)^2}$$

$$\boxed{I_z = \frac{MR^2(15c^4+3H^4+10c^2H^2)}{10(3c^2+H^2)(c^2+H^2)}}$$ en remplaçant ρ par son expression (3.5)

3.8 $c = 0 \longrightarrow \boxed{I_z = \frac{3}{10}MR^2}$ I_z d'un cône, vu en cours

$c \to +\infty \longrightarrow \boxed{I_z \to \frac{MR^2}{2}}$ I_z d'un cylindre