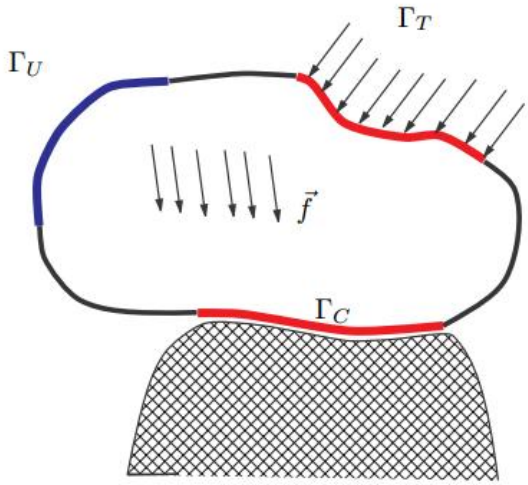
MQ52 - FINAL EXAM

Note: All documents (courses, tutorials and practical exercises) are allowed, except those on electronic devices (laptop, mobile phone …).

The intermediate exam can be written in English or in French as wished by the student.

PROBLEM 1 – CONTACT MECHANICS – 5 POINTS

In this problem, we would like to study the contact problem when an elastic body comes into contact with a rigid, immobile foundation. An elastic body Ω is deformed due to volume and surface forces, but the body should not penetrate a given, rigid obstacle (figure 1): this is the classical Signorini problem.



1. What are the main hypothesizes of the Signorini problem:
   a. Concerning the mechanical situation? Remember quickly the equations of this classical problem.
   b. Concerning the contact conditions?

2. We suppose the normal initial gap equal to zero.
   a. Explain the main steps necessary to write the contact boundary conditions.
   b. What do you remark compared to classical boundary conditions?
   c. Is it a linear or non-linear problem? Justify.
   d. This problem is said non differentiable. Why?

3. Weak form (Variationnal formulation)
   a. Describe and explain the main steps to write the weak form of this contact problem.
   b. What is the main "originality"?

PROBLEM 2 – BIOMECHANICS – 5 POINTS

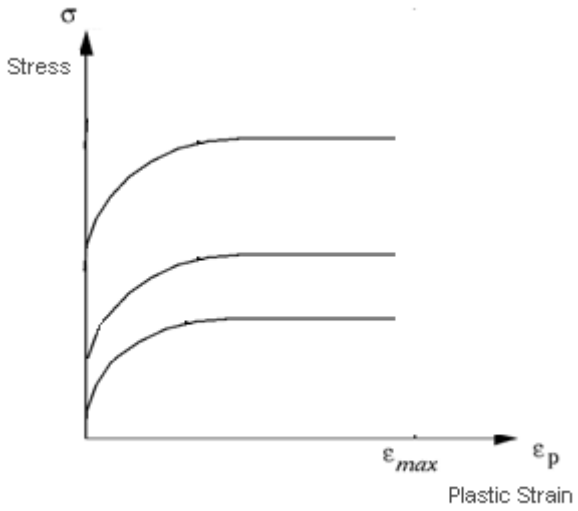
Case study:
In order to improve car seats, engineers would like to use new kind of foams, and would like to make mechanical characterization for an implementation in a FE code for dynamic loading.

- What kind of constitutive law is generally used to model foam material in FE codes?

When there are car accidents, the human body strikes the foam of the car seat.

- What kind of numerical "entity" should you use to simulate the interaction between the human body and the foam? What is the main parameter you have to implement in the FE code for this kind of entity?

Several experimental tests are performed on the foam, named FOAM_A. The different cases are illustrated in the following figure:



- Knowing that the temperature is not taken into account in the experimentations neither in the simulations performed by engineers, at what type of simulations could the different curves of the previous figure correspond (what is the variable in the three curves)?
- According to the previous curves, could the material be considered as a fragile material? A ductile material? Justify your answer.

Engineers want to study the mechanical behavior of the foam until rupture.

- What should be added to the "classical constitutive law" in the simulation in order to study the rupture?

Engineers have decided to simulate the impact of a sphere on a cylinder made of FOAM_A.

- In order to reduce the number of elements and optimize the computational time, what kind of simulation could they perform?

- What kind of errors could occur in the FE simulation, for very large deformation?

Questions:

- For extreme loadings like blast simulation, is the Lagrangian formulation appropriate? Why?
- How can you ensure that a numerical biomechanical model behave like a real physical one?

PROBLEM 3 – MECHANICS – 5 POINTS

Choose a good answer

1. In the geometrically nonlinear analysis, for a **bar** element, the axial strain contains
   A. only the linear terms of the displacement gradients
   B. the linear and quadratic terms of the displacement gradients
   C. the linear and cubic terms of the displacement gradients

2. The approximation of the displacements for a nonlinear **beam** element is
   A. linear
   B. quadratic
   C. cubic

3. In the local coordinate system, a nonlinear **beam** element has
   A. two degrees of freedom
   B. six degrees of freedom
   C. four degrees of freedom

4. In general case, for geometrically nonlinear analysis, the total tangential stiffness matrix is composed of
   A. three matrices
   B. two matrices
   C. one matrix

5. For the initial stability problem or buckling problem, the necessary matrices are the following matrices
   A. initial displacement or large displacement matrix $\mathbf{K_U}$
   B. elastic constant matrix $\mathbf{K_E}$ and initial displacement or large displacement matrix $\mathbf{K_U}$
   C. elastic constant matrix $\mathbf{K_E}$ and initial stress or geometric matrix $\mathbf{K_\sigma}$

PROBLEM 4 – ABRADABLES COATINGS – EFFECTIVE PROPERTIES OF COMPOSITE MATERIALS – 5 POINTS

Generalities:

1. Define in a few words the functionalities of abradable materials in aero engines (**1 pt**)

2. Is it possible to use AlSi/polyester coatings as abradable material in the HP turbine? Please explain why. (**1 pt**)

Estimation of composite coating properties based on their microstructures:

Considering that the stress-strain relationship assumed for a composite coating is:

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{pmatrix} \times \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}$$

in which x stands for the direction parallel to the substrate surface, and y stands for the direction perpendicular to the surface,

3. Please describe a method allowing the estimation of E_x (i.e., type of calculation, boundary conditions, subsequent simplifications and final expression of E_x) (**2 pts**)

4. Please provide in a few words the different calculation steps required in order to estimate ν_{xy} (i.e., sequence of three calculations with their objectives) (**1 pt**)