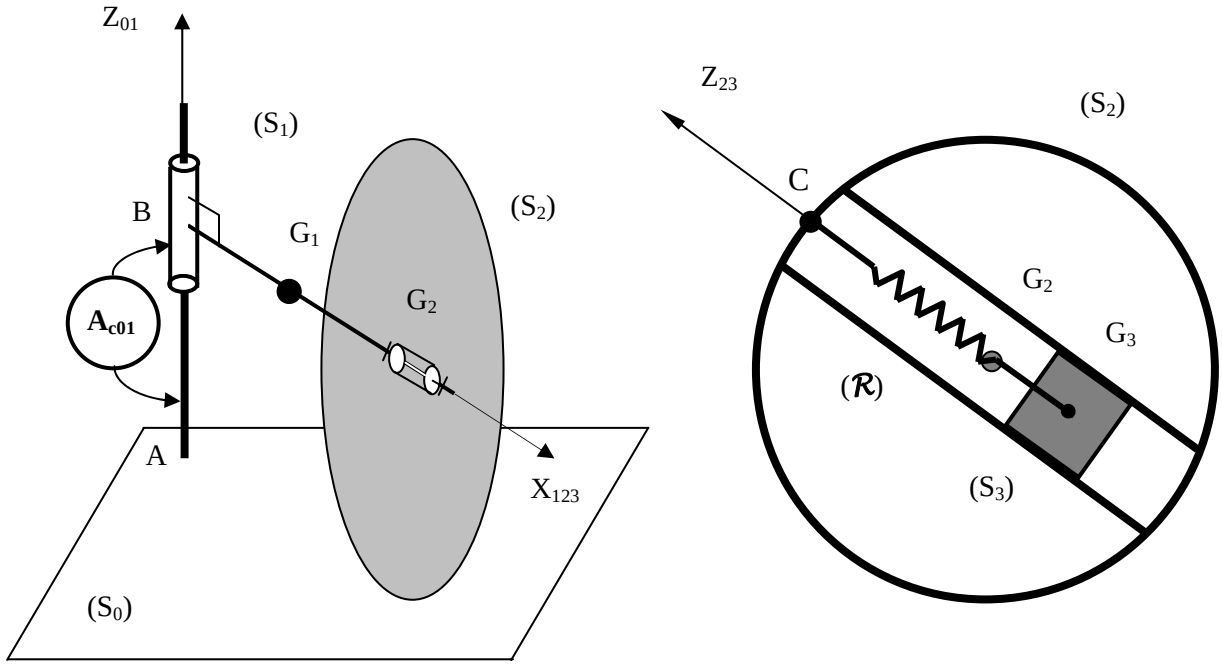
Signature :	***Name :*** ***First name :***	***Mark***	***20***

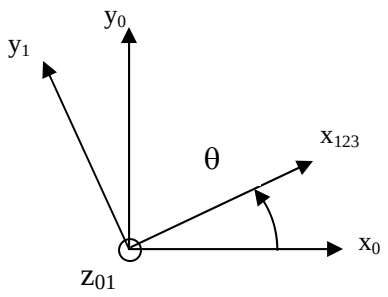
Examination duration: 1h50 – Write your answer to the questions on the examination paper. Only the results are required.

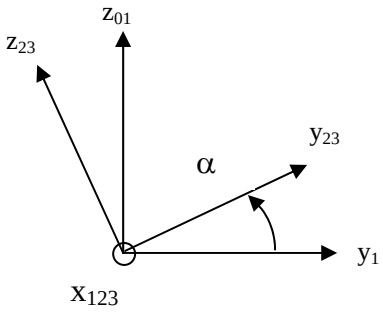
None document authorized.

Apart from this examination paper, no other document should be given back.

The motion of the following device will be studied.







Geometry and distributed mass modelling.

The system identifies :

- four rigid bodies : (S_0), (S_1), (S_2) and (S_3) ;
- a motor (Ac_{01}), considered being external to the study, located in parallel with the cylindrical joint $(S_0 - S_1)$, drives (S_1) relative to (S_0) at a known constant velocity ω_0 ;
- a tensile-compressive spring (R) of negligible mass is located between the bodies (S_2) and (S_3) along a diameter of (S_2).

- six rigid joints.

(S_0 - S_1)	: cylindrical	(S_1 - S_2)	: revolute
(S_2 - S_3)	: prismatic	(S_0 - S_2)	: sphere with a geometric plane of S_0
(R - S_2)	: sphere	(R - S_3)	: sphere located at the centre of mass G_3 of S_3

The centre of mass of (S_2) merged into the centre of the circle and (S_3) is assumed to be a punctual body for the kinetic study.

Forces modelling.

The spring is supposed to be a linear elastic spring.

The cylindrical, revolute, prismatic and spheres are supposed to be perfect joints unlike the sphere plane joint. For this joint, the sliding friction is taken into account through the Coulomb's model.

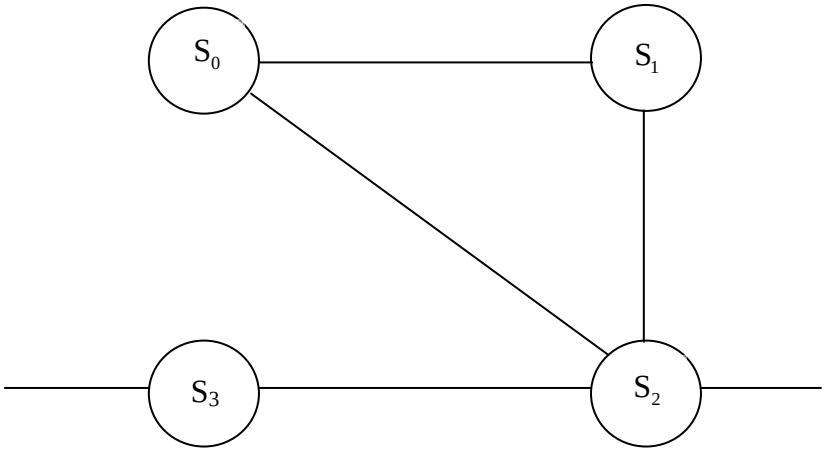
The system moves in the gravitational field which is defined by the vertical unit vector $\vec{z}_{01}$ normal of the plane.

Galilean reference frame.

The fixed body (S_0) frame is supposed to be a Galilean reference frame.

Construct the vectorial geometric model.

Draw the sketch of the joints and write the vectorial models.

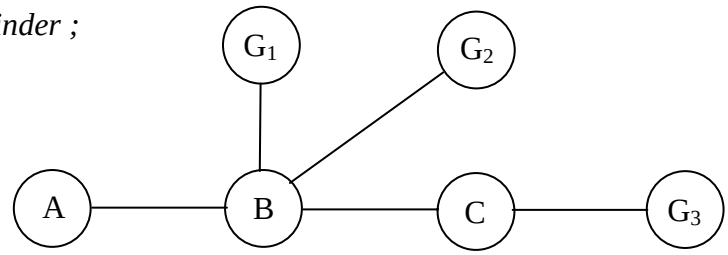


Define the vectorial model of the bodies

$$
\begin{aligned}
R_0 &= R_0 \quad [A \quad ;(\vec{x}_0, \vec{y}_0, \vec{z}_{01})] \\
R_1 &= R_1 \quad [B, G_1 \quad ;(\vec{x}_{123}, \vec{y}_1, \vec{z}_{01})] \\
R_2 &= R_2 \quad [B, C, G_2 \quad ;(\vec{x}_{123}, \vec{y}_{23}, \vec{z}_{23})] \\
R_3 &= R_3 \quad [G_3 \quad ;(\vec{x}_{123}, \vec{y}_{23}, \vec{z}_{23})]
\end{aligned}
$$

Define the parameters :

- the bases : reminder ;
- the points :



$\overrightarrow{AB} = \lambda\ \vec{z}_{01}$ $\overrightarrow{BC} = a\,\vec{x}_{123} + r\,\vec{z}_{23}$ $\overrightarrow{BG_1} = b\ \vec{x}_{123}$ $\overrightarrow{BG_2} = a\ \vec{x}_{123}$ $\overrightarrow{G_3C} = z\,\vec{z}_{23}$

- the equations of constraint ;

- the number of the kinematically independent parameters ;

Construct the vectorial models of the forces and the existence conditions of the joints

a. The spring

b. The sphere plane joint (existence condition and Coulomb's model)

- compute the sliding velocity of (S_2) with respect to (S_0) at the contact point I between the two bodies ;

$\vec{G}_{0,2}(\mathrm{I}) =$

- express the resultant of the twistor of the interaction forces of (S_0) acting on (S_2) ***using its property with respect to the sliding velocity*** *;*

$\vec{S}\ \{S_0 \rightarrow S_2\} =$

- existence condition of the sphere plane joint ;

- Coulomb's law :

- *Rolling without sliding case :*
  - *Existence condition*

  - *Equation*

- *Sliding case :*
  - *Existence condition*

  - *Equation*

c. The zero components of the interaction forces, according to the assumptions of the perfect joints and the rigid bodies

d. The gravitational field

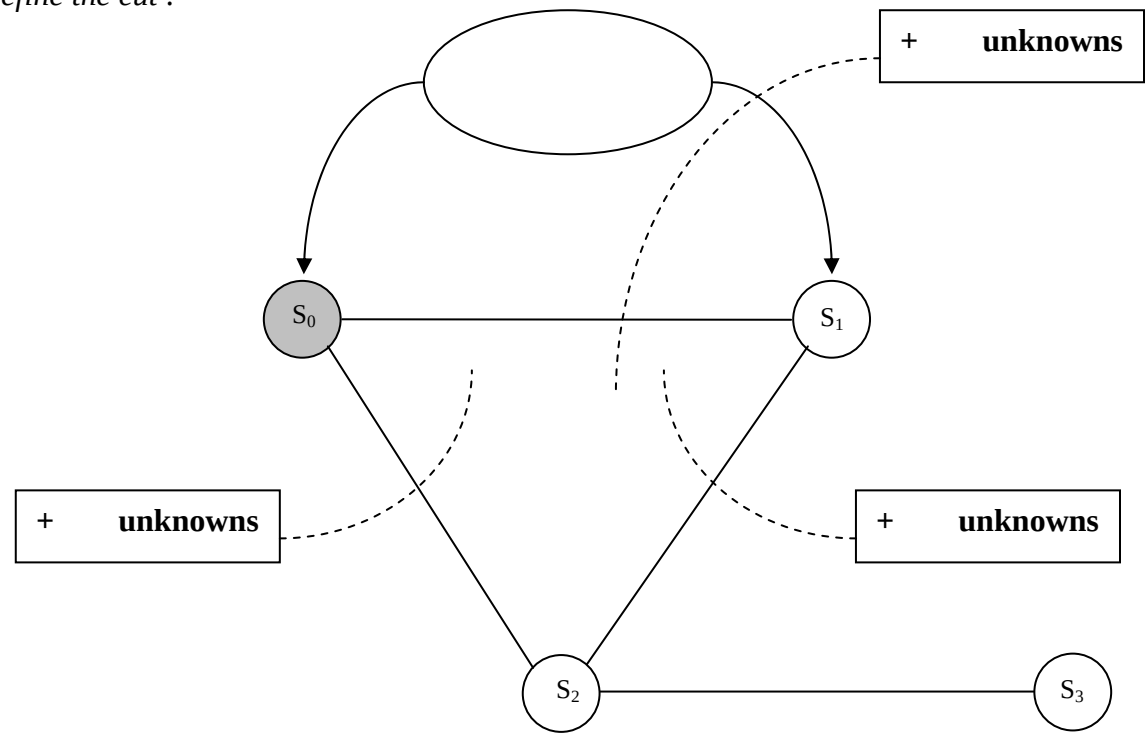
e. The twistor applied by the motor (Ac_{01}), not sought in the study.

$\{Ac_{01} \longrightarrow S_1\} =$

Define the unknowns of the motion study

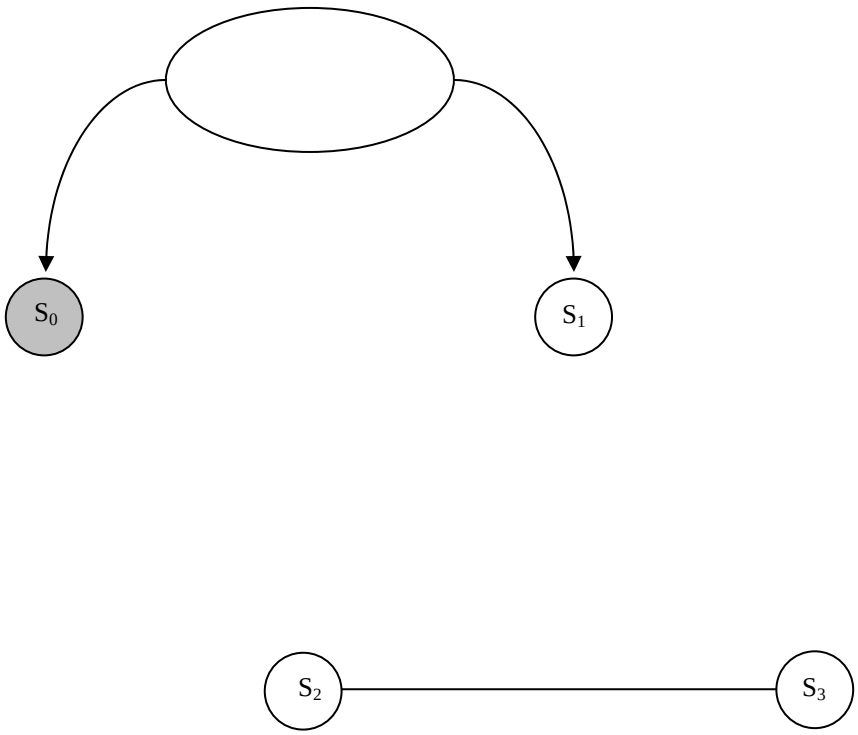
Apply the dynamic theorems

Define the cut :



Define the unknowns of the motion study

Define the sketch of the characteristics :



Write the scalar consequences of the dynamic theorems, ***the angular momentum theorem will be computed at point B*** *which can be considered as a fixed point with respect to the frame* R_0.

Use the scalar consequences of the dynamic theorems

- compute the components of the external forces;

- compute the components of kinetics;