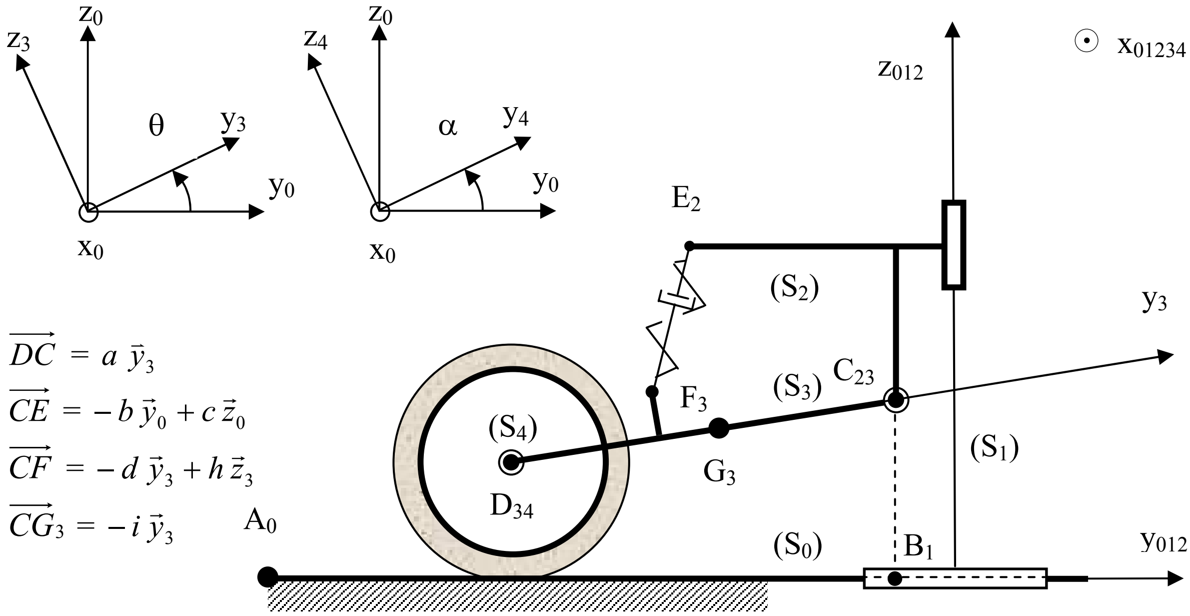
Signature	Name: Firstname:	Mark	/ 20

Examination duration: 1h45 – Write your answer to the questions on the examination paper.
Only the results are required.
Apart from this examination paper, no other document should be given back.

As part of the study of the rear suspension of a scooter, the behaviour of the solution defined in the figure below at the time of acceleration will be studied.



$\overrightarrow{DC} = a\,\vec{y}_3$

$\overrightarrow{CE} = -b\,\vec{y}_0 + c\,\vec{z}_0$

$\overrightarrow{CF} = -d\,\vec{y}_3 + h\,\vec{z}_3$

$\overrightarrow{CG_3} = -i\,\vec{y}_3$

The rear part of the framework of the scooter, loaded with the corresponding mass of the driver, is assumed to be moving in translation with respect to the road. This assumption is represented in the form of an additional intermediate body (S_1), **of negligible mass** located between the road (S_0) and the framework of the scooter (S_2) by means of two prismatic joints.
The transmission casing (S_3) integrates the engine and the revolute joint with the rear wheel (S_4) and realises as well the oscillating arm of the suspension in revolute joint with the framework.

1. The textual dynamic model and the kinematic sketch are given

The device can be modeled as a planar system in the plane of the sketch.

1.1. Geometry and mass

The components of the system are:

- a spring-shock absorber oil-gas (RA) of negligible mass, located between the bodies (S_2) and (S_3);

- 5 rigid bodies:
  - the road (S_0),
  - the intermediate body (S_1),
  - the body (S_2) represents the share of the framework of the scooter fitted out with the accessories and the one of the driver in proportion to the mass divided among the wheels,

- the body (S_3) represents the engine. One neglect the mass of the piston, the connecting road, the crankshaft and all the moving parts of the transmission with respect to the casing,
- the body (S_4) represents the share of the wheel without the tyre,
- the deformation of the tyre on contact with the road is neglected when computing the quantities associated with the mass concept. The wheel, equipped with the tyre, is denoted (S_4^*),
- as far as kinetics is concerned, one assume there is rolling without slipping of the contact point I, located straight up the centre of the wheel, of the tyre with respect to the road ($\vec{V}_{04}(I) \,.\, \vec{y}_0 = 0$);

- 6 rigid joints:

($S_0 - S_1$)	: prismatic	($S_1 - S_2$)	: prismatic
($S_2 - S_3$)	: revolute	($S_3 - S_4$)	: revolute
(Interface RA – S_2)	: sphere	(Interface RA – S_3)	: sphere

1.2. Forces

All the joints between the bodies are assumed to be perfect joints like the two joints at the ends of the spring-shock absorber.

The interaction forces between the road and the tyre are assumed to be expressed by means of a vector passing through the point I, located straight up the centre of the wheel ($\vec{s}_{0,4*} = \mathrm{T}\, \vec{y}_0 + \mathrm{N}\, \vec{z}_0$). The normal component is assumed to be proportional to the crushing of the tyre: N = k_p e with k_p the equivalent stiffness, e the crushing and R the free radius of the tyre.

The engine torque denoted by C_m, defined by the driver, intervenes in parallel with the revolute joint ($S_3 - S_4$).

The magnitude of the actions of air, the drag force, on the couple scooter-driver can be expressed as $\frac{1}{2}\rho C_x S V^2$:

- C_x: drag coefficient
- ρ: mass density of air
- S: reference area
- V: longitudinal velocity of the scooter

The system is moving in the gravitational field defined by the vertical $\vec{z}_0$.

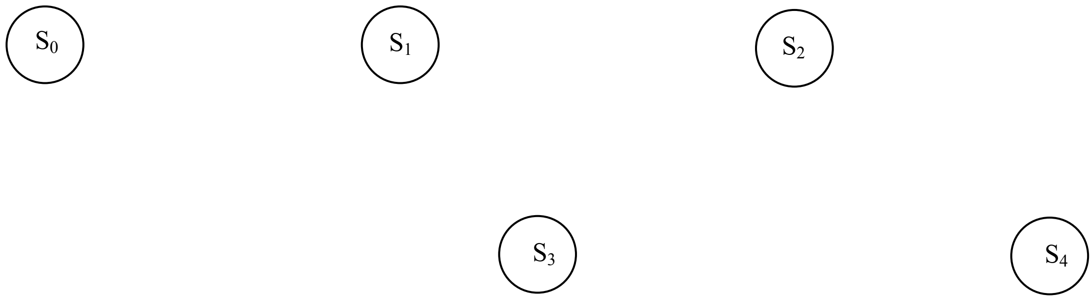
1.3. Galilean frame of reference

The frame attached to the body (S_0) is assumed to be galilean.

2. Construct the vectorial geometric model

2.1. Define the vector models of the joints

- draw the sketch of the joints
- identify the fixed geometric elements and the geometric conditions of the joints on the figure below.

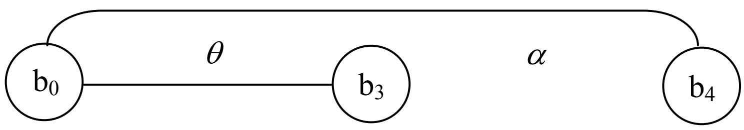


2.2. Define the vector models of the bodies

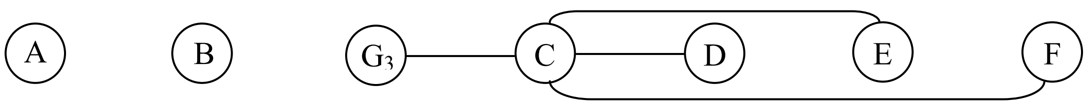
$$R_0 = R_0 \left[\quad ;(\vec{x} \quad ,\vec{y} \quad ,\vec{z} \quad)\right]$$
$$R_1 = R_1 \left[\quad ;(\vec{x} \quad ,\vec{y} \quad ,\vec{z} \quad)\right]$$
$$R_2 = R_2 \left[\quad ;(\vec{x} \quad ,\vec{y} \quad ,\vec{z} \quad)\right]$$
$$R_3 = R_3 \left[\quad ;(\vec{x} \quad ,\vec{y} \quad ,\vec{z} \quad)\right]$$
$$R_4 = R_4 \left[\quad ;(\vec{x} \quad ,\vec{y} \quad ,\vec{z} \quad)\right]$$

2.3. Define the parameters

- use a minimum path between the bases



- use a minimum path between the points



=   =

2.4. Define the equations of constraint

- use the joints not yet taken into account

- use the characteristics of some of the joints not yet taken into account

$\vec{V}_{04}(I) \, . \, \vec{y}_0 = 0 \Rightarrow$

- take into account the laws of behaviour of the motors

2.5. Identify the kinematically independent parameters

3. Express the laws of behaviour in terms of the geometric model

3.1. The spring- shock absorber (RA)

$\{(RA) \rightarrow S_2\} = -\{(RA) \rightarrow S_3\}$

$\vec{S}\,\{(RA) \rightarrow S_3\} = \lambda(\theta, \theta')\overrightarrow{FE}$

$\vec{M}_F\{(RA) \rightarrow S_3\} = \vec{0}, \vec{M}_E\{(RA) \rightarrow S_2\} = \vec{0}$

Clarify solely the vector below in order to compute later the complementary term in the relation of the vector moment field.

$\overrightarrow{FE} =$

3.2. The action of air

$\vec{S}\,\{(Air) \rightarrow S_2\} =$

3.3. The engine torque

$\vec{M}_D\{(Engine\ torque) \rightarrow S_4\} =$

3.4. The action of the road on the tyre

- $\vec{s}_{0,4^*} = T\,\vec{y}_0 + N\,\vec{z}_0$ with $N = k_p\, e$ (e the crushing and R the free radius of the tyre)

e =

- condition of the existence of the action of the road on the tyre

3.5. The zero components of the interaction forces between the bodies

= =

= =

3.6. The gravitational field

4. Define the unknowns of the study

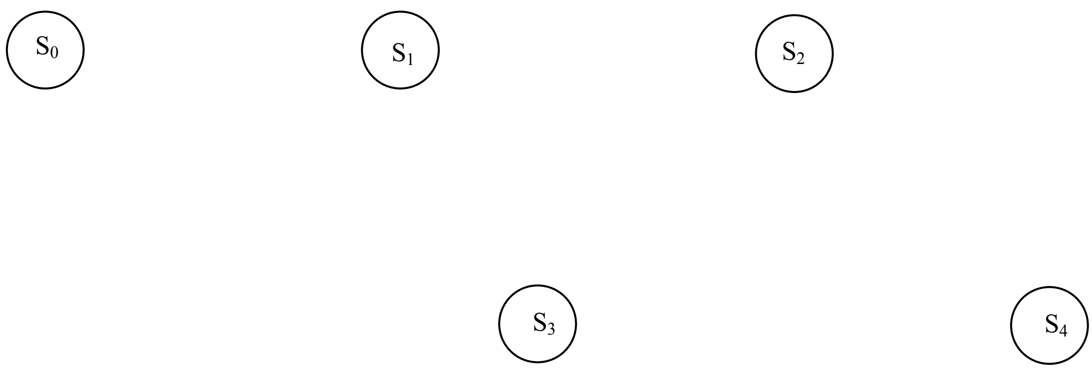
The study concerns the motion and the search of the coefficient of friction to install in order to guarantee the rolling without slipping between the road and the tyre at the contact point I.

5. Write the dynamic equations

In the following, one retains the initial parameters.

5.1. Is there a closed-loop?

5.2. Draw the sketch of the characteristics



5.3. Write the scalar consequences of the dynamic theorems

(1) = *with E =*

(2) = *with E =*

(3) = *with E =*

(4) = *with E =*

5.4. Compute the components of the external forces

(1) =

(2) =

(3) =

(4) =

5.5. Compute the components of kinetics

(1) =

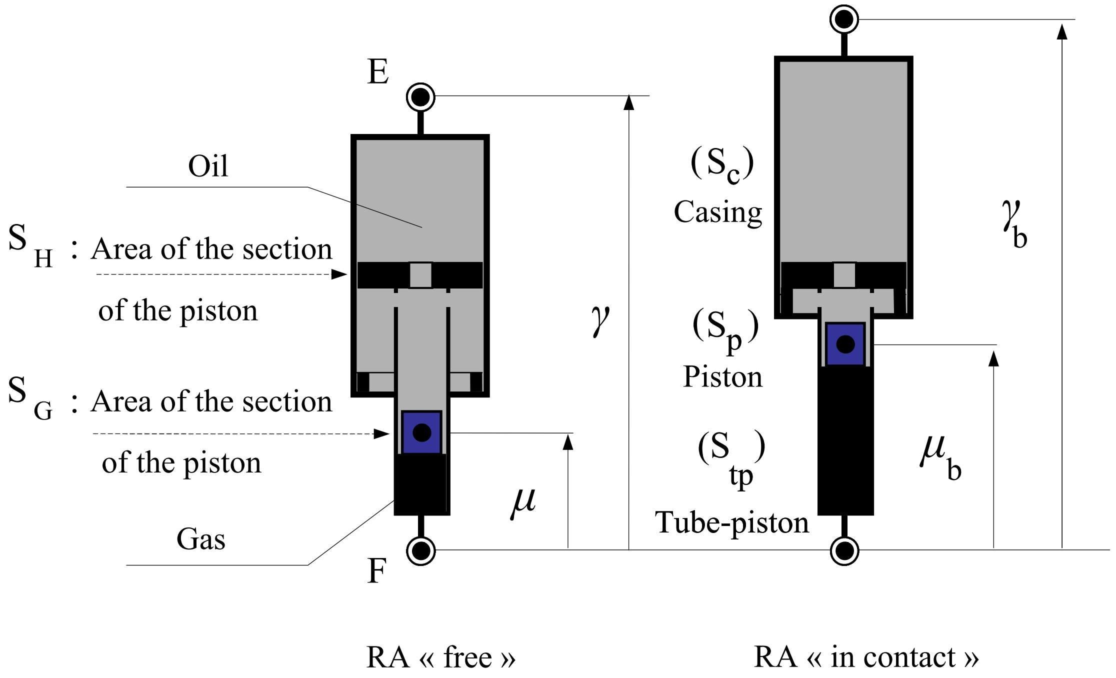
(2) =

(3) =

(4) =

6. Complete the study of the spring-shock absorber

Use the notations defined on the figure below.



Within the spring-shock absorber:

- the joints are assumed to be perfect;
- the actions of the oil pressure are assumed to be uniform inside each chamber:
  - the flow of the oil through the calibrated passage in the tube-piston establishes a difference of pressure between the two chambers (upper s and lower i) containing it;
  - the difference of pressure is assumed to be proportional to the velocity of the tube-piston (tp) inside the casing (c) with η the constant of proportionality;
- the pressure actions due to the gas fit with a reversible adiabatic process:

pression * volume ^{k} = cste = $p_b V_b^k$ with p_b et V_b given, the pressure and the volume of the gas when the spring-shock absorber is in contact.

6.1. Spring-shock absorber « free »

- which is the condition of the existence of the step RA "free"?

- define the equation of constraint between μ and γ, in terms of the assumption "non compressible oil" and neglecting the thickness of the wall of the tube-piston

γ =

 M. Ferney, M. Meyer and D. Sauhet

- express the action of the spring-shock absorber on the body (S_3)

$\vec{S}\,\{(\text{RA}) \to S_3\} =$

6.2. Spring-shock absorber « in contact »

- which is the condition of the existence of the step RA "in contact"?

- define the equation of constraint

- express the action of the spring-shock absorber on the body (S_3) assuming the area of the contact between the piston and the thrust is negligible

$\vec{S}\,\{(\text{RA}) \to S_3\} =$