Signature	Name: First Name:	

Examination duration: 2h – Write your answer to the questions on the examination paper.
Except this examination paper, no other document should be given back.

All the results will be proved

No document allowed - SMARTPHONES, GSM and Calculators prohibited.

I- First part

Known-how in mechanics, answer the following questions.

In dynamics, in the case of a partial search for unknowns, list the options provided by the principle of dynamics application in order to obtain the set of equations limited to the unknowns of the study?

$$\{D_{gE}\}=\{\overline{E}\rightarrow E\}$$

a:

b:

c:

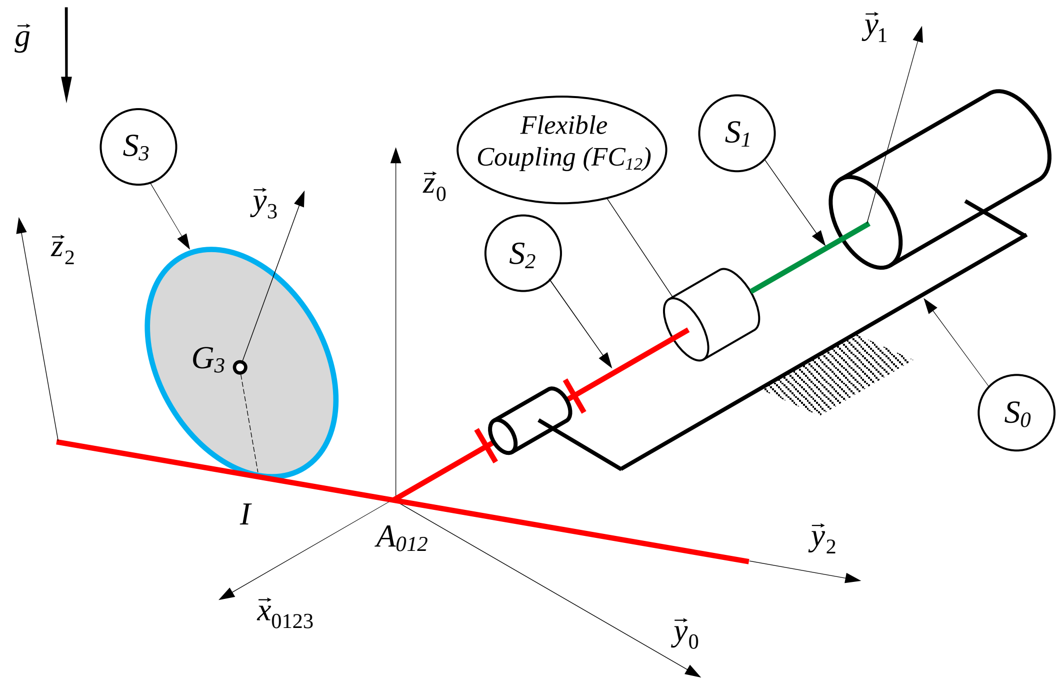
d:

List the various cases of a perfect point surface joint by direct contact.

M. Meyer, D. Sauhet

II- Second part

The ***motion*** of the experimental device shown below and the ***condition of maintaining contact*** between the bodies (S_2) and (S_3) will be studied.



1. Textual dynamic model and kinematic sketch

1.1. Geometry and mass

The system identifies:

- a driving motor *(DM_{01})* causing its output shaft (S_1) an harmonic motion with respect to its stator − body (S_0) – defined by $\theta = \theta_M \sin(\omega t)$.

- a linear elastic flexible coupling *(FC_{12})*, located between the bodies (S_1) and (S_2), of negligible mass and torsional stiffness k_T. The components of the interaction forces twistor of *(S_1)* acting upon *(S_2)* are:

$$\{S_1 \rightarrow S_2\} = \begin{Bmatrix} \vec{0} \\ -k_T(\alpha - \theta)\vec{x}_0 \end{Bmatrix}$$

knowing that at the initial instant all the parameters have a null value.

- four rigid bodies: (S_0), (S_1), (S_2) and (S_3):
  - (S_0) the ground, stator of the driving motor *(DM_{01})*,
  - (S_1) the output shaft of the driving motor *(DM_{01})*,
  - (S_2) of centre of inertia G_2 which coincides with A_{012} and of mass m_2,
  - (S_3) a body of revolution, of radius R, of centre of inertia G_3 and of mass m_3,

- Three rigid joints:

  ($S_0 - S_1$) revolute *(not shown in kinematic sketch)*

  ($S_0 - S_2$) revolute ($S_2 - S_3$) point surface

1.2. Forces

The revolute joints are supposed to be perfect joints unlike the one sided point surface. For this joint, the sliding friction is taken into account through the Coulomb's model.

The study can be modeled as a planar motion in the plane $\left(A_{012}, \vec{y}_0, \vec{z}_0\right)$

The device moves in the gravitational field which is defined by the vertically upward direction $\vec{z}_0$.

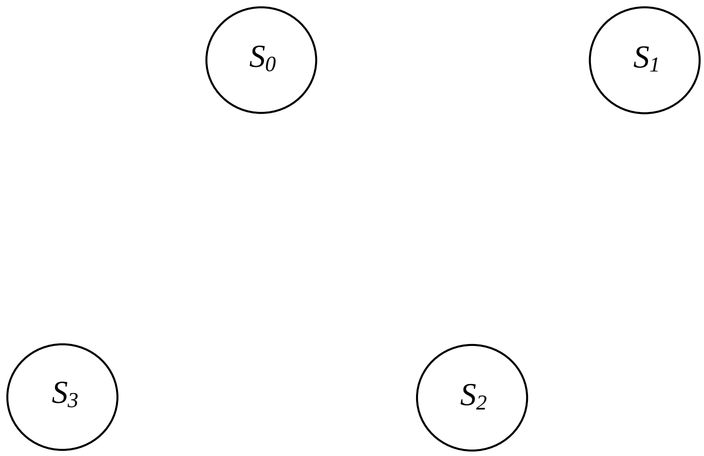
1.3. Galilean reference frame

In the field of the study, the fixed body *(S_0)* is supposed to be a Galilean reference frame.

2. Construct a vector geometric model

2.1. Model the joints

- draw the sketch of the joints



2.2. Model the rigid bodies

$$R_0 = R_0 \left[\qquad ; \left(\vec{x} \quad , \vec{y} \quad , \vec{z} \quad \right) \right]$$

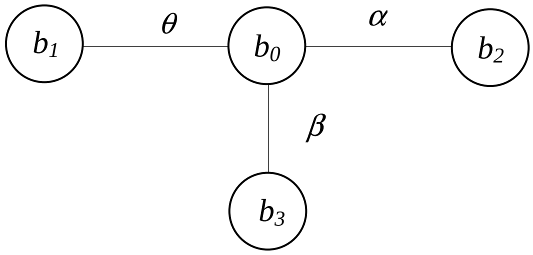
$$R_1 = R_1 \left[\qquad ; \left(\vec{x} \quad , \vec{y} \quad , \vec{z} \quad \right) \right]$$

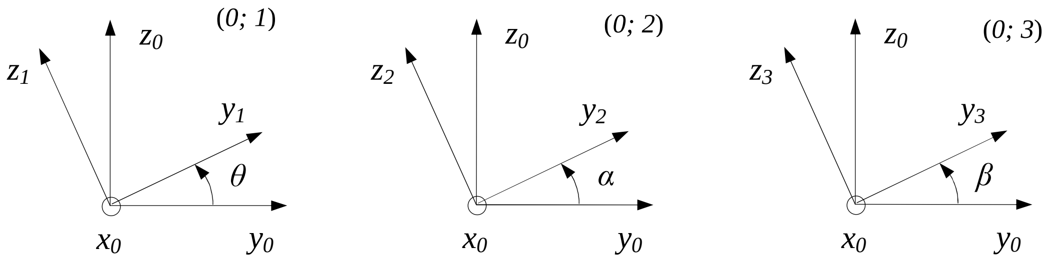
$$R_2 = R_2 \left[\qquad ; \left(\vec{x} \quad , \vec{y} \quad , \vec{z} \quad \right) \right]$$

$$R_3 = R_3 \left[\qquad ; \left(\vec{x} \quad , \vec{y} \quad , \vec{z} \quad \right) \right]$$

2.3. Parameterize

- use a minimum path between the bases *(for information)*





- use a minimum path between the points

$\overrightarrow{AG_3} =$

2.4. Study the equations of constraint

- use the joints not taken yet into account through their vector model

Consequence of the one sided point surface ($S_2 - S_3$)

- use the geometric condition of the joints not taken yet into account

- take into account the laws of behaviour of the driving motor

2.5. Define the kinematically independent parameters

- Number of kinematically independent parameters: *Number =*
- Define the kinematically independent parameters:

3. Express the laws of behaviour in terms of the geometric model

- *the flexible coupling (FC_{12}):*

- *the sliding velocity between (S_3) and (S_2) at the contact point I*

$$\vec{G}_{23}(I) =$$

- *the components of the resultant of the interaction forces twistor of (S_2) acting upon (S_3)*

$$\vec{s}\{S_2 \rightarrow S_3\} =$$

- *the condition of existence of the one sided point surface joint*

- *the Coulomb's law*
  - *Case of rolling without sliding*
    - *Condition of existence:*
    - *Equation:*
  - *Case of sliding*
    - *Condition of existence:*
    - *Equation:*

- *the zero components of the perfect joints between the rigid bodies*

- *the interaction forces twistor of the driving motor (DM_{01}) acting upon (S_1), unsearched in this study*

$$\{DM_{01} \rightarrow S_1\} =$$

- *the gravitational field*

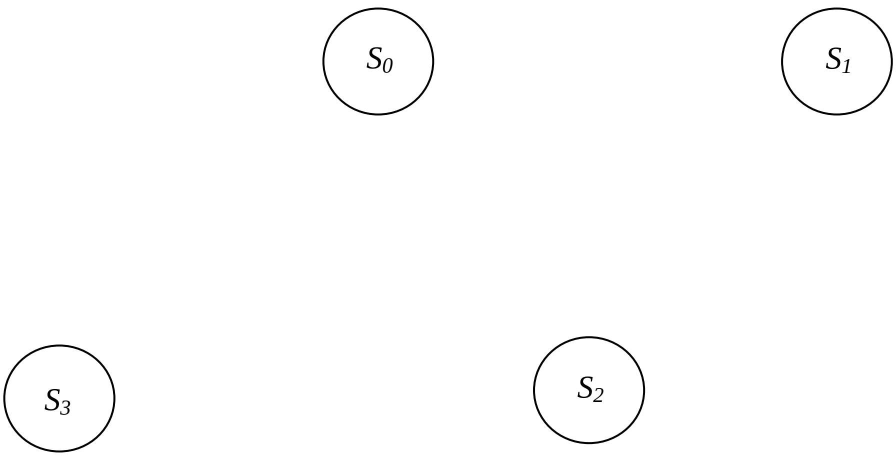
4. Gather the unknowns of the study

- *the kinematically independent parameters:*
- *the components of the interaction forces introduced from the laws of behaviour:*
- *the components of the interaction forces unknowns of the study:*

5. Write the equations of dynamics

5.1. Is there a closed loop?

5.2. Draw the sketch of the characteristics



5.3. Write the scalar consequences of the dynamic equations

= *with E =*

= *with E =*

= *with E =*

= *with E =*

5.4. Compute the components of forces

=

=

=

=

5.5. Compute the components of kinetics

- *Compute the dynamic resultant of (S_3) in its motion with respect to (S_0)*

$$\vec{s}\{D_{0,3}\}=$$

- Compute the dynamic moments about the point A of (S_2) and (S_3) in their motion with respect to (S_0)

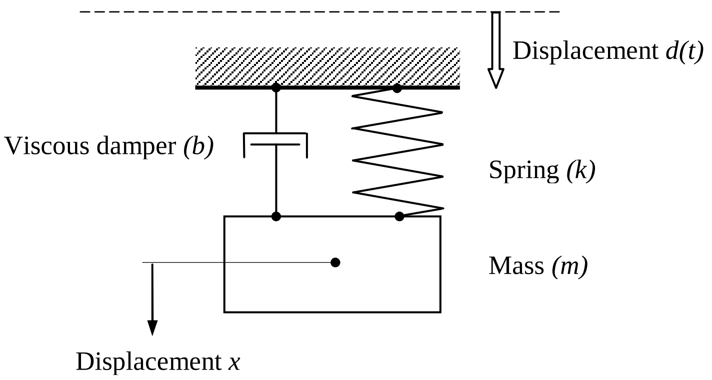
$$\vec{\delta}_{0,2\cup 3}(A)=$$

- Compute the dynamic moment about the point I of (S_3) in its motion with respect to (S_0)

$$\vec{\delta}_{0,3}(I)=$$

III- Third part

Study, with SIMULINK, the motion of a viscously damped spring-mass system subjected to the displacement *d(t)* of the support as shown in the figure below.



Build, page 10, a Simulink model that solves the following equation of motion of the system:

$$m\ddot{x} + b\dot{x} + kx = k\,d(t) + b\,\dot{d}(t)$$

The variable *x* denotes the displacement of the mass from its equilibrium position.

Given informations:

- *m* = 20 kg: mass of the system,
- *k* = 17,5 kN/m: spring stiffness,
- *b* = 240 N.s/m: viscous damping coefficient,
- *d(t)* = 50 *sin* (10*t*) mm: displacement of the support.

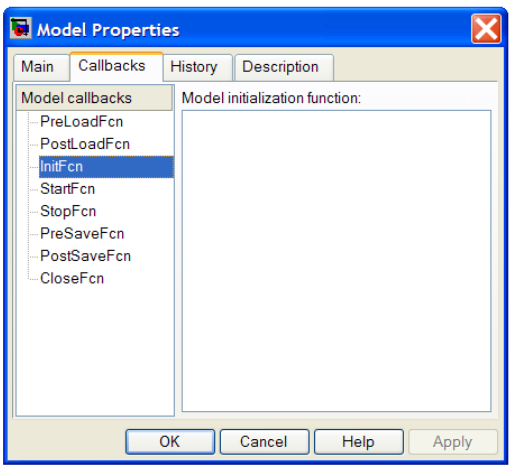
Initial conditions:

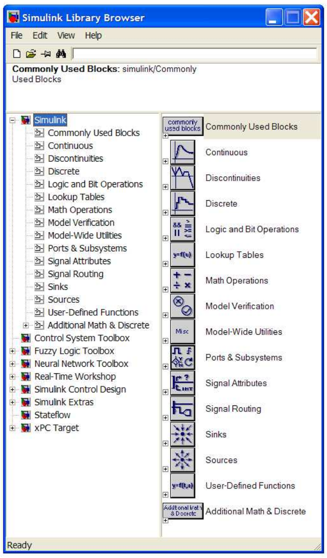
- $x(t = 0) = x_0 = 0$,
- $x'(t = 0) = x'_0 = 0$.

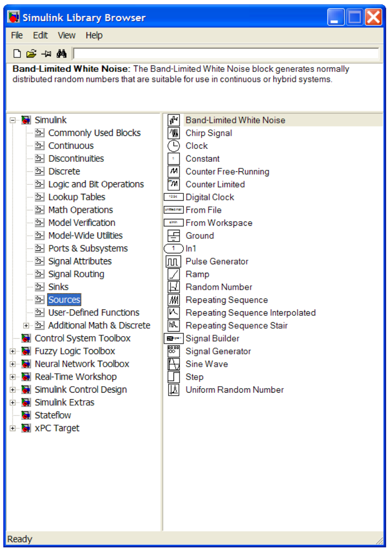
Display of results:
The displacement *x* will be displayed as:

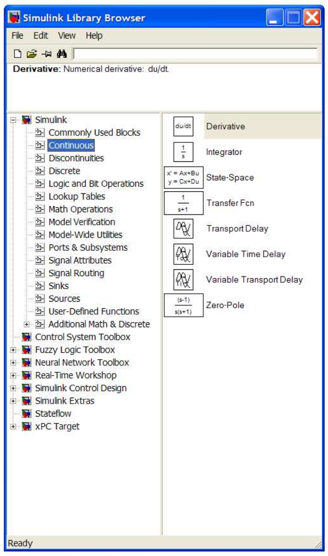
- a curve *x* = *x(t)*,
- a matrix *x* = *x(t)*.
-

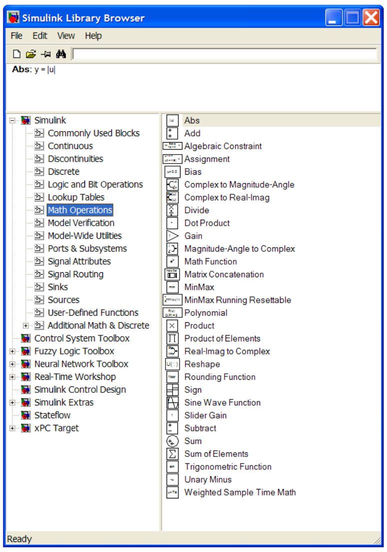
SIMULINK model:

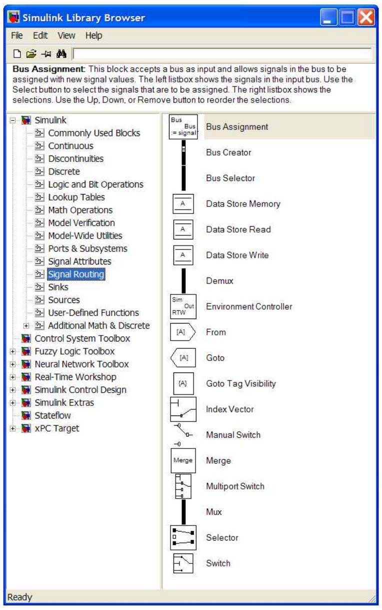


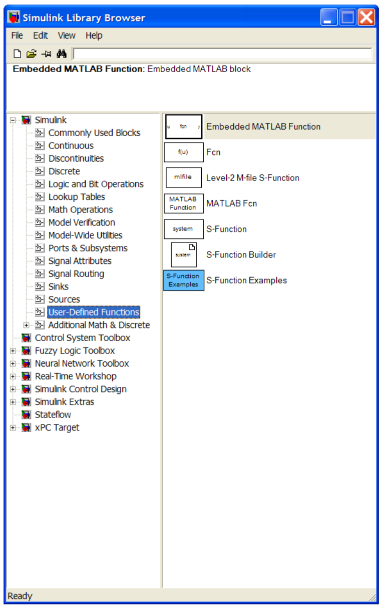

Simulink Library Browser
File Edit View Help
Commonly Used Blocks: simulink/Commonly Used Blocks
Simulink
Commonly Used Blocks
Continuous
Discontinuities
Discrete
Logic and Bit Operations
Lookup Tables
Math Operations
Model Verification
Model-Wide Utilities
Ports & Subsystems
Signal Attributes
Signal Routing
Sinks
Sources
User-Defined Functions
Additional Math & Discrete
Control System Toolbox
Fuzzy Logic Toolbox
Neural Network Toolbox
Real-Time Workshop
Simulink Control Design
Simulink Extras
Stateflow
xPC Target
Commonly Used Blocks
Continuous
Discontinuities
Discrete
Logic and Bit Operations
Lookup Tables
Math Operations
Model Verification
Model-Wide Utilities
Ports & Subsystems
Signal Attributes
Signal Routing
Sinks
Sources
User-Defined Functions
Additional Math & Discrete
Ready

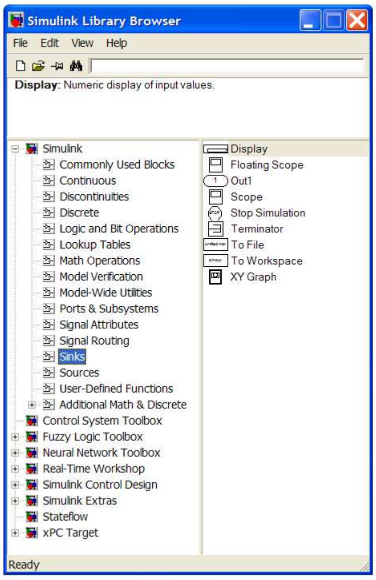

Simulink Library Browser
File Edit View Help
Band-Limited White Noise: The Band-Limited White Noise block generates normally distributed random numbers that are suitable for use in continuous or hybrid systems.
Band-Limited White Noise
Chirp Signal
Clock
Constant
Counter Free-Running
Counter Limited
Digital Clock
From File
From Workspace
Ground
In1
Pulse Generator
Ramp
Random Number
Repeating Sequence
Repeating Sequence Interpolated
Repeating Sequence Stair
Signal Builder
Signal Generator
Sine Wave
Step
Uniform Random Number
Ready


Simulink Library Browser
File Edit View Help
Derivative: Numerical derivative: du/dt
Derivative
Integrator
State-Space
Transfer Fcn
Transport Delay
Variable Time Delay
Variable Transport Delay
Zero-Pole
Ready


Simulink Library Browser
File Edit View Help
Abs: y = |u|
Abs
Add
Algebraic Constraint
Assignment
Bias
Complex to Magnitude-Angle
Complex to Real-Imag
Divide
Dot Product
Gain
Magnitude-Angle to Complex
Math Function
Matrix Concatenation
MinMax
MinMax Running Resettable
Polynomial
Product
Product of Elements
Real-Imag to Complex
Reshape
Rounding Function
Sign
Sine Wave Function
Slider Gain
Subtract
Sum
Sum of Elements
Trigonometric Function
Unary Minus
Weighted Sample Time Math
Ready


Simulink Library Browser
File Edit View Help
Bus Assignment: This block accepts a bus as input and allows signals in the bus to be assigned with new signal values. The left listbox shows the signals in the input bus. Use the Select button to select the signals that are to be assigned. The right listbox shows the selections. Use the Up, Down, or Remove button to reorder the selections.
Bus Assignment
Bus Creator
Bus Selector
Data Store Memory
Data Store Read
Data Store Write
Demux
Environment Controller
From
Goto
Goto Tag Visibility
Index Vector
Manual Switch
Merge
Multiport Switch
Mux
Selector
Switch
Ready


Simulink Library Browser
File Edit View Help
Embedded MATLAB Function: Embedded MATLAB block
Embedded MATLAB Function
Fcn
Level-2 M-file S-Function
MATLAB Fcn
S-Function
S-Function Builder
S-Function Examples
Ready


Simulink Library Browser
File Edit View Help
Display: Numeric display of input values.
Display
Floating Scope
Out1
Scope
Stop Simulation
Terminator
To File
To Workspace
XY Graph
Ready